Matrix Theory — Exam 2
MAT 335, Fall 2022 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (12pts) Matrix A is given below.
a) Evaluate its determinant by any (efficient) method.
b) State if A is invertible and justify.

$$\begin{vmatrix} 1 & 3 & 0 & 0 \\ -1 & 1 & 2 & -3 \\ 2 & 6 & 3 & 0 \\ 0 & 4 & 2 & 1 \end{vmatrix} =$$

2. (6pts) If A and B are 3×3 matrices with $\det A = 3$ and $\det B = -2$, find:

$\det(A^{-1}B) =$

$\det(2A) =$

$\det(-A^T B) =$

3. (12pts) The matrix A is given below.
a) Find the inverse of A.
b) Use the inverse to easily solve the system below.

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$\begin{aligned} 3x_1 \; &+5x_2 &= -2 \\ x_1 \; &+2x_2 &= 1 \end{aligned}$$

4. (14pts) Find the standard matrix of the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ and determine whether T is a) one-to-one, or b) onto.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - x_2 + 4x_3 \\ 2x_1 + x_2 + x_3 \\ 3x_1 + 9x_2 - 6x_3 \end{bmatrix}$$

5. (8pts) For a function $T : \mathbf{R}^2 \to \mathbf{R}^3$ the following is known:
a) T is a linear transformation
b) $T(\mathbf{e}_2) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ and $T\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$.

Find the standard matrix of T.

6. (16pts) A matrix A is given below.
a) Find a basis for the nullspace of A.
b) Find a basis for the column space of A.

$$A = \begin{bmatrix} 1 & 4 & 1 & -1 \\ -2 & -7 & 0 & -1 \\ 1 & 5 & 3 & -3 \end{bmatrix}$$

7. (14pts) The set W is defined below.
a) Use the definition to show W is a subspace of $\mathbf{R}^3$.
b) Give a set of generating vectors for W.

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbf{R}^3 \mid x_1 + 2x_2 = 0 \text{ and } x_2 - x_3 = 0 \right\}$$

8. (18pts) Are the following statements true or false? Justify your answer by giving a logical argument or a counterexample.

a) If $\{\mathbf{u}_1, \mathbf{u}_2\}$ is linearly independent and T is a *nonzero* linear transformation, then $\{T(\mathbf{u}_1), T(\mathbf{u}_2)\}$ is linearly independent.

b) For a 2×2 matrix A, if $\det A = 0$, then the columns of A are parallel.

c) The set $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbf{R}^2 \mid x_1^2 + x_2^2 \geq 1 \right\}$ is a subspace of $\mathbf{R}^2$.

Bonus. (10pts) Let A be an $n \times n$ matrix with columns $\mathbf{a}_1, \ldots, \mathbf{a}_n$. If B is the matrix with columns $\mathbf{a}_2, \mathbf{a}_3, \ldots, \mathbf{a}_n, \mathbf{a}_1$ (in that order), how are $\det A$ and $\det B$ related?