

Matrix Theory — Exam 1
MAT 335, Fall 2022 — D. Ivanišić

Name: _____
Show all your work!

1. (12pts) For the matrices A , B and C find the following expressions, if they are defined:
a) AA^T b) BA c) $CB - B$

$$A = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 5 \\ 2 & -1 \end{bmatrix}$$

2. (8pts) The augmented matrix below is in reduced row echelon form.
a) Write the solution of the system represented by the matrix.
b) Write the solution in vector form.

$$A = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 3 & 4 \end{array} \right]$$

3. (8pts) Write the rotation matrix for a counterclockwise rotation around the origin by angle $\frac{2\pi}{3}$ and use it find where the point $(2, 1)$ lands after it is rotated.

4. (14pts) A system of linear equations is given below.

a) Use the Gaussian elimination to solve the system.

b) Write the solution in vector form.

$$\begin{cases} 3x_1 - 2x_2 - 3x_3 - 15x_4 = -9 \\ x_1 - x_2 - 2x_3 - 8x_4 = -3 \\ -2x_1 + x_2 + 2x_3 + 9x_4 = 5 \end{cases}$$

5. (10pts) Below is the input-output matrix for an economy producing food, transportation and tourism.

a) What net production corresponds to a gross production of \$30M of food, \$50M of transportation and \$40M of tourism?

b) Set up the system (write its augmented matrix) to find the gross production needed to satisfy exactly demand of \$35M of food, \$20M of transportation and \$50M of tourism. Do NOT solve the system, or you will suffer.

food	transp.	tourism	
0.2	0.2	0.3	food
0.3	0.1	0.3	transportation
0.1	0.1	0.1	tourism

6. (12pts) Below is the augmented matrix of a system of linear equations. Determine the coefficients a and b for which the system has: a) one solution, b) infinitely many solutions, c) no solutions. (Note: no row operations are needed.)

$$A = \left[\begin{array}{ccc|c} 1 & 4 & -5 & 0 \\ 0 & a & -2 & 3 \\ 0 & 0 & b+3 & 7 \end{array} \right]$$

7. (6pts) Find the elementary matrix E so that $EA = B$.

$$A = \begin{bmatrix} 0 & -2 & 7 \\ 3 & 2 & -1 \\ -4 & 6 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -2 & 7 \\ -4 & 6 & 0 \end{bmatrix} \quad E =$$

8. (12pts) Consider the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ -2 \\ -3 \end{bmatrix}$.

a) Do they span \mathbf{R}^4 ?

b) Are they linearly independent?

9. (18pts) Are the following statements true or false? Justify your answer by giving a logical argument or a counterexample.

a) If \mathbf{a} and \mathbf{b} are in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$, then $\mathbf{a} - 3\mathbf{b}$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$.

b) If A is a 3×3 matrix with rank 2, then the columns of A are linearly dependent.

c) If 2×2 matrices A and B are invertible, then $A + B$ is invertible.

Bonus. (10pts) Show: if vectors \mathbf{u} and \mathbf{v} are linearly independent, then vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are linearly independent.