Calculus 2 — Exam 0
MAT 308, Spring 2026 — D. Ivanšić

Name: ____________________

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\frac{d}{dx}\left(5x^9 - \pi^3 - 3\sqrt[6]{x^{11}} + \frac{5}{x^2}\right) =$

2. (6pts) $\frac{d}{dx}\left(\sqrt{x}e^{3x+2}\right) =$

3. (8pts) $\frac{d}{du}\frac{5u^4}{\left(u^2+2u+1\right)^3} =$

4. (4pts) $\frac{d}{dx}\frac{1}{(\cos x - \sin x)^2} =$

5. (6pts) $\frac{d}{dx}\arctan(\ln(x^3 - 2x)) =$

6. (7pts) Find the first and second derivatives of $f(x) = \tan\left(\sqrt{x}\right)$.

7. (5pts) Let $f(x) = x \ln x$. Take the first four derivatives of f, and try to spot the pattern. What is $f^{(29)}(x)$, the 29th derivative of f? How about $f^{(n)}(x)$?

Find the following limits. Use L'Hospital's rule if needed.

8. (4pts) $\lim_{x \to 3+} \frac{1}{x-3} =$

9. (5pts) $\lim_{x \to \infty} \frac{x^2 - 6x + 1}{3x^4 + x^2} =$

10. (7pts) $\lim_{x \to 0} x^4 \ln |x| =$

Find the following antiderivatives.

11. (7pts) $\displaystyle\int 5x^5 + \frac{3}{\sqrt{1-x^2}} + \sqrt[4]{x^9} + a^3\, dx =$

12. (3pts) $\displaystyle\int \tan\left(2x + \frac{\pi}{3}\right)\, dx =$

13. (7pts) $\displaystyle\int x^6(\sqrt{x} + x\sqrt[3]{x})\, dx =$

Use the substitution rule in the following integrals:

14. (7pts) $\displaystyle\int \sec^2 x\, \sqrt[5]{\tan x}\, dx =$

15. (10pts) $\displaystyle\int_{\sqrt{\frac{3}{2}}}^{\sqrt{3}} \frac{x\, dx}{\sqrt{1 - \left(\frac{x^2}{3}\right)^2}} =$

16. (8pts) Consider the integral $\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x \, dx$.

a) Draw a picture and use the "area" interpretation to explain the what the integral represents.

b) Use the picture to estimate whether the integral is positive or negative.

c) Evaluate the integral to verify your finding in b).

Bonus. (10pts) The rear inside cover of our book claims that

$$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

Verify this formula by differentiating.

Calculus 2 — Exam 1 **MAT 308, Spring 2026 — D. Ivanšić**

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Show all your work!

Find the following integrals:

1. (10pts) $\int x^2 e^x \, dx =$

2. (8pts) $\int \sin^2 x \, \cos^5 x \, dx =$

Determine whether the following improper integral converges by calculating it directly.

3. (14pts) $\int_1^\infty \frac{\ln x}{x^3} \, dx =$

Use trigonometric substitution to evaluate the following integrals. Don't forget to return to the original variable where appropriate.

4. (12pts) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-1}}\,dx =$

5. (14pts) $\displaystyle\int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx =$

Use the method of partial fractions to find the integral.

6. (14pts) $\displaystyle\int \frac{6x^2 + x + 33}{(2x+3)(x^2+9)}\,dx =$

7. (10pts) Use comparison to determine whether the improper integral $\displaystyle\int_1^{\infty} \frac{x^2}{x^4+6}\,dx$ converges.

8. (18pts) Suppose we wanted to approximate the number $\ln 2$. We could do it by approximating the integral $\int_{\frac{1}{2}}^{1} \frac{1}{x}\,dx = \ln 2$, which uses only the four algebraic operations.
a) Write the expression you would use to calculate M_4, the midpoint rule with 4 subintervals. All the terms need to be explicitly written, do not use f in the sum and do not simplify.
b) Find the error estimate for M_n in general. You will need the second derivative of $\frac{1}{x}$.
c) Estimate the error for M_4.
d) What should n be in order for M_n to give you an error less than 10^{-4}?

Bonus (10pts) Show the reduction formula.

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

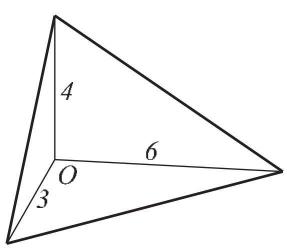
Calculus 2 — Exam 2 **MAT 308, Spring 2026 — D. Ivanšić**	**Name:** ____________________ *Show all your work!*

1. (24pts) The region bounded by the curves $y = e^x$, $y = x + 1$ and $x = 2$ is rotated around the y-axis. (Note that $y = e^x$ and $y = x + 1$ have only one common point.)
a) Sketch the solid and a typical cylindrical shell.
b) Set up the integral for the volume of the solid using the shell method.
c) Evaluate the integral.

2. (14pts) Consider the half of the region bounded by the curves $y = x^2$, $y = \frac{1}{3}x^2$ and $y = 5$ that is in the first quadrant.
a) Sketch the region.
b) Set up the integral that computes its area. Simplify, but do not evaluate the integral.

3. (14pts) Rotate the region bounded by the curves $y = x^2 + 4x + 6$ and $y = 36 - x^2$ about the x-axis to get a solid.
a) Sketch the solid and a typical cross-sectional washer.
b) Set up the integral for the volume of the solid. Do NOT simplify, and do not evaluate the integral.

4. (18pts) At point O, the sides of the tetrahedron shown below all meet at right angles.
a) Devise a way to take cross-sections of this solid and draw two of them.
b) Determine the area of each cross-section as a function of a suitable variable. Similar triangles will be useful.
c) Set up the integral for the volume of the tetrahedron. Simplify, but do not evaluate the integral.



5. (16pts) Find the length of the curve $y = \ln \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$.

6. (14pts) Set up the integral to find the work expended when five people take the elevator from the first to the sixth floor of Faculty Hall. Assume $g = 10$, the empty elevator weighs 150kg, the five people together weigh 320kg, each floor has height 3m and the cable lifting the elevator weighs 4kg per meter. Simplify, but do not evaluate the integral.

Bonus (10pts) If we rotate the curve $y = x^2$ around the y-axis, we get a surface called a *paraboloid*. Compute the surface area of the part of the paraboloid that comes from rotating the section of $y = x^2$ for $0 \leq x \leq 3$.

Calculus 2 — Exam 3
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Show all your work!

Find the limits, if they exist.

1. (6pts) $\lim_{n\to\infty} \dfrac{3^{2n+4}}{2^{3n-1}} =$

2. (6pts) $\lim_{n\to\infty} \sin\dfrac{(2n+1)\pi}{2} =$

3. (10pts) Find the limit. Use the theorem that rhymes with what you would unlock a door with.

$$\lim_{n\to\infty} \frac{\cos(n^2-1)}{\sqrt{n}+\sqrt[3]{n}}$$

4. (6pts) Write the series using sigma notation:

$$\frac{2}{1\cdot 2} - \frac{5}{1\cdot 2\cdot 3\cdot 4} + \frac{8}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} - \frac{11}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8} + \cdots =$$

5. (12pts) Justify why the series converges and find its sum.

$$\sum_{n=2}^{\infty} \frac{2^{2n-1}}{7^{n-2}} =$$

Determine whether the following series converge and justify your answer.

6. (6pts) $\displaystyle\sum_{n=1}^{\infty} \frac{1-n^2}{n^2+5}$

7. (12pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4+17}{n^6-n}$

8. (20pts) Consider the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n+7}$.

a) Is the series convergent? Justify.
b) Is the series absolutely convergent? Justify.
c) How many terms of the series do we need to add to find the sum with accuracy 0.1? (If you find the inequality too hard to solve, try some numbers that are easy to compute.)

Determine whether the following series converge using the root or ratio test.

9. (11pts) $\sum_{n=1}^{\infty}\frac{2^{n+5}(\arctan n)^n}{3^{n+1}(n+7)}$

10. (11pts) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n(4n+1)}{n!}$

Bonus. (10pts) Write the following infinite repeating decimal number as a fraction with integers in numerator and denominator.

$0.24\,561\,561\,561\cdots =$

Calculus 2 — Exam 4
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Show all your work!

Find the intervals of convergence for the series below. Don't forget to check the endpoints.

1. (10pts) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{10^n}{(2n)!}\, x^n$

2. (16pts) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n\cdot 3^{n+1}}\,(x-2)^n$

3. (6pts) Use a known power series to find the sum.

$$\sum_{n=0}^{\infty} \frac{(\ln 2)^{n-1}}{n!} =$$

4. (8pts) Use known power series to find the limit.

$$\lim_{x \to 0} \frac{\sin x - x \cos x}{x^3} =$$

5. (14pts) Use geometric series to get a power series for $\dfrac{x^3}{6+x^2}$. Your answer needs to be a single sum of type $\sum c_n x^n$. State the interval of convergence (no need to check the endpoints).

6. (14pts) Recall that $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-\frac{1}{2}}$.
a) Use a binomial series and antidifferentiation to find the McLaurin series for $\arcsin x$. Do NOT expand the binomial coefficient $\binom{k}{n}$ in the general sum.
b) Write the first four terms of the series in a), this time expanding the binomial coefficients and simplifying.

7. (16pts) Let $f(x) = \cos x$.
a) Find the 3rd Taylor polynomial for f centered at $a = \pi$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$. Leave your answer as a fraction.

8. (16pts) Use the known McLaurin series for $\ln(1+x)$ to find the series representing $\int_0^1 \ln(1+x^2)\,dx$. Give an approximation of this definite integral with accuracy 10^{-2}. Write the approximation as a sum (you do not have to simplify it).

Bonus (10pts) Let $a_n =$ the n-th digit of π, so $a_1 = 3$, $a_2 = 1$, $a_3 = 4$, etc. Find the interval of convergence for the series below. Don't forget to check the endpoints.

$$\sum_{n=1}^{\infty} \frac{a_n + 1}{2^n} x^n = \frac{4}{2}x + \frac{2}{4}x^2 + \frac{5}{8}x^3 + \dots$$

Calculus 2 — Exam 5
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Name: ______________________________

Show all your work!

1. (12pts) Polar coordinates of two points are given.
a) Sketch the points in the plane.
b) For each point, give two additional polar coordinates, one with a negative r, one with a negative θ.

$\left(4, \dfrac{2\pi}{3}\right)$ $\left(-2, \dfrac{3\pi}{7}\right)$

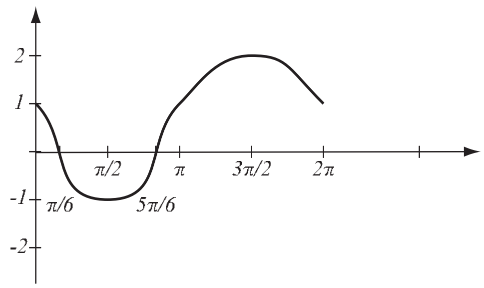
2. (10pts) Convert (a picture may help):
a) $\left(3, \dfrac{7\pi}{4}\right)$ from polar to rectangular coordinates
b) $(-3\sqrt{3}, -9)$ from rectangular to polar coordinates

3. (14pts) Find the equation of the tangent line to the parametric curve $x = te^t$, $y = te^{-t}$ at the point where $t = 2$.

4. (12pts) A particle moves along the path with parametric equations $x(t) = 1 + 3\sin t$, $y(t) = -2 + 3\cos t$ for $0 \leq t \leq 3\pi$. Sketch the path of motion and then describe the motion of the particle.

5. (8pts) Identify the curve given in polar coordinates by $r = 2\cos\theta + 2\sin\theta$ by converting the equation to cartesian coordinates.

6. (12pts) The graph of $r = f(\theta)$ is given in cartesian coordinates. Use its intervals of increase and decrease to help you sketch the polar curve $r = f(\theta)$. Indicate which piece of the cartesian graph corresponds to which piece of the polar graph.



7. (14pts) The parametric curve $x = \frac{t^2}{2}$, $y = \frac{1}{9}(6t+9)^{\frac{3}{2}}$, $-1 \leq t \leq 3$ is given.
Use an integral to find the length of the curve.

8. (18pts) A parametric curve is given by $x(t) = t^3 - 3t$, $y(t) = -t^3 + 12t$.
a) Find the points on the curve where the tangent line is horizontal or vertical.
b) Where does the curve go as $t \to \infty$ and $t \to -\infty$? (That is, find $\lim_{t \to \pm\infty} x(t)$, $\lim_{t \to \pm\infty} y(t)$.)
c) Plot the points from a) on a coordinate system and use them, along with information from b), or from plotting additional points, to get a graph of the curve. Recall that the curve moves in only one of general directions $\nearrow \nwarrow \swarrow \searrow$ between points from a).

Bonus. (10pts) The parametric curve $x = 2\cos^2 t$, $y = 2\sin t \cos t$, $0 \leq t \leq 2\pi$ is given. Eliminate the parameter to get the equation of the curve in cartesian coordinates. Then describe the how the curve is being traced by the parametrization. *(Hint: double-angle formulas for sine and cosine are useful.)*

Calculus 2 — Final Exam
MAT 308, Spring 2026 — D. Ivanšić

Name: ______________________

Show all your work!

Find the following integrals:

1. (6pts) $\int x \ln x \, dx =$

2. (10pts) $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx =$

3. (12pts) Determine whether the following improper integral converges by calculating it directly.

$$\int_0^{\infty} x e^{-x} \, dx =$$

4. (10pts) Convert (a picture may help):

a) $\left(8, \dfrac{7\pi}{4}\right)$ from polar to rectangular coordinates

b) $(-2\sqrt{3}, 6)$ from rectangular to polar coordinates

5. (24pts) The region bounded by the curves $y = 2 + \sqrt{x}$ and $y = \dfrac{x}{2} + 2$ is rotated around the x-axis.

a) Sketch the solid and a typical cross-sectional washer.

b) Set up the integral for the volume of the solid.

c) On another picture, sketch the solid and a typical cylindrical shell.

d) Set up the integral for the volume of the solid using the shell method.

Simplify, but do not evaluate the integrals.

6. (10pts) Justify why the series converges and find its sum.

$$\sum_{n=2}^{\infty}(-1)^n \frac{6 \cdot 7^n}{2^{3n+2}} =$$

7. (14pts) Find the interval of convergence of the series. Don't forget to check the endpoints.

$$\sum_{n=1}^{\infty} \frac{2^{n+2}(x-4)^n}{\sqrt{n}}$$

8. (18pts) Let $f(x) = \sqrt[3]{x}$.
a) Find the 3rd Taylor polynomial for f centered at $a = 8$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $(6, 10)$.

9. (10pts) A particle moves along the path with parametric equations $x(t) = e^t + 1$, $y(t) = e^{2t} - 1$, t any. Eliminate the parameter in order to sketch the path of motion and then describe the motion of the particle.

10. (24pts) The integral $\int_0^1 e^{-x^2}\,dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of $f(x) = e^{-x^2}$ is not expressible using elementary functions.
a) Write the expression you would use to calculate M_6, the midpoint rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) It is known that $-2 < f''(x) < \frac{4}{5}$ on $[0, 1]$: use it to find the error estimate for M_n in general.
c) What should n be in order for M_n to give you an error less than 10^{-4}?
d) Use the known power series for e^x to find a power series for the above integral.
e) How many terms of the power series are needed to estimate the integral to accuracy 10^{-4}? Write the estimate as a sum (you do not have to simplify it).
f) Which method requires less computation to evaluate the integral with accuracy 10^{-4}, Simpson rule or series?

11. (12pts) First draw the graph of $r = \cos(2\theta)$ in a cartesian θ-r coordinates. Use this graph to draw the polar curve with the same equation.

Bonus (15pts) Show the reduction formula.

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$