Calculus 2 — Final Exam
MAT 308, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

Find the following integrals:

1. (6pts) $\int xe^{2x}\,dx =$

2. (10pts) $\int_0^{\frac{\pi}{2}} \cos^3 x \sin^3 x\,dx =$

3. (12pts) Determine whether the following improper integral converges by calculating it directly.

$\int_1^{\infty} \frac{\ln x}{x^2}\,dx =$

4. (10pts) Convert (a picture may help):
a) $\left(4, \frac{5\pi}{4}\right)$ from polar to rectangular coordinates
b) $(3, -3\sqrt{3})$ from rectangular to polar coordinates

5. (24pts) The region bounded by the curves $y = x^2 + 1$ and $y = 5$ is rotated around the x-axis.
a) Sketch the solid and a typical cross-sectional washer.
b) Set up the integral for the volume of the solid.
c) On another picture, sketch the solid and a typical cylindrical shell.
d) Set up the integral for the volume of the solid using the shell method.
Simplify, but do not evaluate the integrals.

6. (10pts) Justify why the series converges and find its sum.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5\cdot 3^{2n+1}}{16^n} =$$

7. (14pts) Find the interval of convergence of the series. Don't forget to check the endpoints.

$$\sum_{n=1}^{\infty}\frac{(x-2)^n}{4^{n+1}\cdot(n+3)}$$

8. (18pts) Let $f(x) = \sqrt{x}$.
a) Find the 3rd Taylor polynomial for f centered at $a = 9$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $(6, 12)$.

9. (10pts) A particle moves along the path with parametric equations $x(t) = 3 + \sin^2 t$, $y(t) = 1 - \cos^2 t$, $0 \leq t \leq \pi$. Eliminate the parameter in order to sketch the path of motion and then describe the motion of the particle.

10. (24pts) The integral $\int_0^1 \sin(x^2)\,dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of $f(x) = \sin(x^2)$ is not expressible using elementary functions.
a) Write the expression you would use to calculate S_6, the Simpson rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) It is known that $-29 < f^{(4)}(x) < 0$ on $[0, 1]$: use it to find the error estimate for S_n in general.
c) What should n be in order for S_n to give you an error less than 10^{-4}?
d) Use the known power series for $\sin x$ to find a power series for the above integral.
e) How many terms of the power series are needed to estimate the integral to accuracy 10^{-4}? Write the estimate as a sum (you do not have to simplify it).
f) Which method requires less computation to evaluate the integral with accuracy 10^{-4}, Simpson rule or series?

11. (12pts) First draw the graph of $r = \cos\theta - \frac{1}{2}$ in a cartesian θ-r coordinates. Use this graph to draw the polar curve with the same equation.

Bonus (15pts) Find a fraction that is the approximation of e with accuracy 10^{-4}. Use the series for e^x and Taylor's formula, and assume you know $e < 3$. Write the approximation as a sum (you do not have to simplify it).