Calculus 2 — Exam 4 **MAT 308, Fall 2021 — D. Ivanšić**

Name: ______________________________

Show all your work!

Find the intervals of convergence for the series below. Don't forget to check the endpoints.

1. (16pts) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n n^2} \cdot (x-1)^n$

2. (10pts) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)} \, x^n$

3. (6pts) Use a known power series to find the sum. It's not a typo — it really is $(2n)!$ in the denominator, not $(2n+1)!$ Think.

$$\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{3^{2n+1}(2n)!} =$$

4. (8pts) Use a known power series to find the limit.

$$\lim_{x \to 0} \frac{\ln(1+3x^3) - 3x^3}{x^6} =$$

5. (14pts) Use geometric series to get a power series for $\dfrac{2x-8}{x^2-8x+15}$. The partial fraction decomposition has been written for you. Your answer needs to be a single sum of type $\sum c_n x^n$. State the interval of convergence (no need to check the endpoints).

$$\frac{2x-8}{x^2-8x+15} = \frac{1}{x-5} + \frac{1}{x-3} =$$

6. (12pts) Use a geometric series and antidifferentiation to find the McLaurin series for $\arctan x$.

7. (18pts) Let $f(x) = \ln x$.
a) Find the 3nd Taylor polynomial for f centered at $a = 4$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $[3, 5]$. Leave your answer as a fraction.

8. (16pts) Use the known power series for $\sin x$ to find the series representing $\int_0^1 \sin(x^2)\,dx$. (Note that $\sin(x^2)$ does not have an antiderivative that is an elementary function.) Give an approximation of the definite integral with accuracy 10^{-4}. Write the approximation as a sum (you do not have to simplify it).

Bonus (10pts) Find a fraction that is the approximation of e with accuracy 10^{-3}. Use the series for e^x and Taylor's formula, and assume you know $e < 3$. Write the approximation as a sum (you do not have to simplify it).