Calculus 2 — Exam 3
MAT 308, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

Find the limits, if they exist.

1. (6pts) $\lim_{n\to\infty} (-1)^n \frac{2^{2n+3}}{5^n} =$

2. (6pts) $\lim_{n\to\infty} \tan \frac{(4n+1)\pi}{4} =$

3. (10pts) Find the limit. Use the theorem that rhymes with the blank from "_____ Navidad."

$$\lim_{n\to\infty} \frac{5+(-1)^n}{n^4+3n^2}$$

4. (6pts) Write the series using sigma notation:

$$\frac{7}{5} - \frac{9}{25} + \frac{11}{125} - \frac{13}{625} + \cdots =$$

5. (12pts) Justify why the series converges and find its sum.

$$\sum_{n=1}^{\infty} \frac{2^{3n}}{5 \cdot 3^{2n+3}} =$$

Determine whether the following series converge and justify your answer.

6. (12pts) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^2+5n-3}$

7. (6pts) $\displaystyle\sum_{n=1}^{\infty} \left(\sin \frac{1}{n} - \sqrt[n]{2} \right)$

8. (8pts) Consider the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$.

a) Is the series convergent? Justify.

b) Is the series absolutely convergent? Justify.

9. (12pts) For the sequence $\left\{\frac{n}{(n+5)^2}\right\}_{n=1}^{\infty}$, determine:

a) for which n the sequence is decreasing.

b) its upper and lower bounds.

Determine whether the following series converge using the root or ratio test.

10. (11pts) $\sum_{n=2}^{\infty}\frac{2^{2n}(n^2+n+7)}{3^{n+1}(n^3-n^2)}$

11. (11pts) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^{11}}{4^n \cdot n!}$

Bonus. (10pts) Consider the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ below.

a) Show that $\lim_{n\to\infty} b_n = 0$ by considering the odd and even terms separately.

b) Show that the sequence b_n is not decreasing.

c) Show that the partial sum s_{2n} of the series below satisfies $s_{2n} = u_n - v_n$, where u_n, v_n are partial sums of familiar series.

d) Use c) to help you answer: does the series below converge?

$$\frac{1}{1} - \frac{1}{2^1} + \frac{1}{2} - \frac{1}{2^2} + \frac{1}{3} - \frac{1}{2^3} + \frac{1}{4} - \frac{1}{2^4} + \cdots =$$