Calculus 2 — Exam 1
MAT 308, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

Find the following integrals:

1. (12pts) $\int e^x \cos x \, dx =$

2. (8pts) $\int \sin^2 x \, \cos^2 x \, dx =$

Determine whether the following improper integral converges by calculating it directly.

3. (10pts) $\int_0^\infty \frac{\arctan x}{1+x^2} \, dx =$

Use trigonometric substitution to evaluate the following integrals. Don't forget to return to the original variable where appropriate.

4. (14pts) $\int x^3 \sqrt{1-x^2}\, dx =$

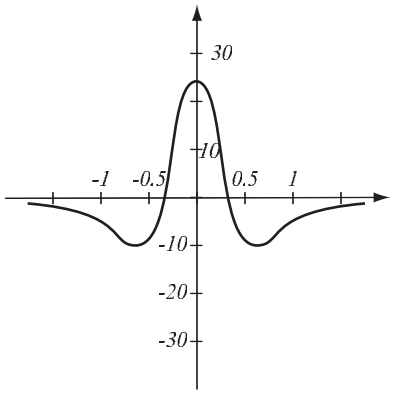
5. (14pts) $\int_0^3 \frac{1}{(9+x^2)^{\frac{3}{2}}}\, dx =$

Use the method of partial fractions to find the integral.

6. (14pts) $\displaystyle\int \frac{-x^2+2x+3}{(x-1)^3}\,dx =$

7. (10pts) Use comparison to determine whether the improper integral $\displaystyle\int_2^\infty \frac{x^5}{x^6-4}\,dx$ converges.

8. (18pts) Suppose we wanted to approximate the number $\frac{\pi}{4} = \arctan 1$. We could do it by approximating the integral $\int_0^1 \frac{1}{1+x^2}\,dx = \frac{\pi}{4}$, which uses only the four algebraic operations.
a) Write the expression you would use to calculate S_6, the Simpson rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) Find the error estimate for S_n in general. The graph of the fourth derivative of $\frac{1}{1+x^2}$, which is $\frac{24(5x^4-10x^2+1)}{(1+x^2)^5}$, is shown in the picture.
c) Estimate the error for S_6.
d) What should n be in order for S_n to give you an error less than 10^{-8}?



Bonus (10pts) Determine for which $p > 0$ the integral below converges. (Note this is not the standard knowledge one because the interval is different.)

$$\int_0^1 \frac{1}{x^p}\,dx$$