Calculus 2 — Exam 0
MAT 308, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(3x^4 - e^6 + \sqrt[3]{x^5} + \dfrac{4}{x^7}\right) =$

2. (6pts) $\dfrac{d}{dx}\left(e^{2x}\cos^3 x\right) =$

3. (8pts) $\dfrac{d}{dt}\dfrac{\left(t^2-1\right)^3}{\left(t^2+1\right)^2} =$

4. (4pts) $\dfrac{d}{dx}\dfrac{1}{x\ln x} =$

5. (6pts) $\dfrac{d}{d\theta}\arcsin^2(\tan\theta) =$

6. (7pts) Find the first and second derivatives of $f(x) = \cos\left(x^2\right)$.

7. (5pts) Let $f(x) = \dfrac{1}{x^3}$. Take the first four derivatives of f, and try to spot the pattern. What is $f^{(20)}(x)$, the 20th derivative of f? How about $f^{(n)}(x)$?

Find the following limits. Use L'Hospital's rule if needed.

8. (2pts) $\lim\limits_{x\to 0-} \dfrac{1}{x^5} =$

9. (6pts) $\lim\limits_{x\to\infty} \dfrac{x^3 - 5x + 4}{-3x^2 + x + 2} =$

10. (8pts) $\lim\limits_{x\to\infty} (1 + \ln x)^{\frac{1}{x}} =$

Find the following antiderivatives.

11. (7pts) $\int 2x^4 - \dfrac{2}{1+x^2} + \sqrt[5]{x^6} + c^2\, dx =$

12. (3pts) $\int \sin\left(4x - \dfrac{\pi}{4}\right)\, dx =$

13. (7pts) $\int \dfrac{\sqrt{x} + \sqrt[3]{x}}{\sqrt[6]{x}}\, dx =$

Use the substitution rule in the following integrals:

14. (7pts) $\int \dfrac{e^x}{1+e^{2x}}\, dx =$

15. (10pts) $\displaystyle\int_{e^8}^{e^{64}} \frac{dx}{x\sqrt[3]{\ln x}} =$

16. (8pts) Consider the integral $\int_{\frac{\pi}{6}}^{\frac{3\pi}{4}} \cos x \, dx$.

a) Draw a picture to explain the meaning of the integral.
b) Use the picture to estimate whether the integral is positive or negative.
c) Evaluate the integral to verify your finding in b).

Bonus. (10pts) The rear inside cover of our book claims that

$$\int \frac{dx}{x^2 - a^2} \, dx = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C$$

Verify this formula by differentiating.