Calculus 2 — Exam 5
MAT 308, Spring 2026 — D. Ivanšić

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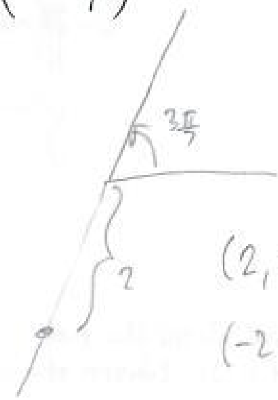
1. (12pts) Polar coordinates of two points are given.
a) Sketch the points in the plane.
b) For each point, give two additional polar coordinates, one with a negative r, one with a negative θ.

$\left(4, \frac{2\pi}{3}\right)$

$\left(-4, -\frac{\pi}{3}\right)$

$\left(4, -\frac{4\pi}{3}\right)$

$\left(-2, \frac{3\pi}{7}\right)$



$\left(2, -\frac{4\pi}{7}\right)$

$\left(-2, -\frac{11\pi}{7}\right)$

2. (10pts) Convert (a picture may help):
a) $\left(3, \frac{7\pi}{4}\right)$ from polar to rectangular coordinates
b) $(-3\sqrt{3}, -9)$ from rectangular to polar coordinates

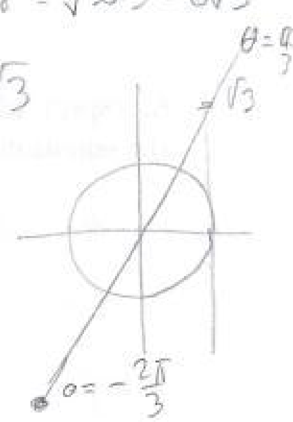
a) $x = 3\cos\frac{7\pi}{4} = 3\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$

$y = 3\sin\frac{7\pi}{4} = 3\left(-\frac{\sqrt{2}}{2}\right) = -\frac{3\sqrt{2}}{2}$

b) $r = \sqrt{(-3\sqrt{3})^2 + (-9)^2}$

$= \sqrt{27+81} = \sqrt{108} = \sqrt{36\cdot 3} = 6\sqrt{3}$

$\tan\theta = \frac{-9}{-3\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$



$\theta = -\frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$\left(6\sqrt{3}, -\frac{2\pi}{3}\right)$

3. (14pts) Find the equation of the tangent line to the parametric curve $x = te^t$, $y = te^{-t}$ at the point where $t = 2$.

$x' = e^t + te^t = (1+t)e^t \quad x'(2) = 3e^2 \qquad x(2) = 2e^2$

$y' = e^{-t} + te^{-t}(-1) = (1-t)e^{-t} \quad y'(2) = -e^{-2} = -\frac{1}{e^2} \qquad y(2) = \frac{2}{e^2}$

$\frac{y'}{x'} = \frac{-\frac{1}{e^2}}{3e^2} = -\frac{1}{3e^4}$

Tan line: $y - \frac{2}{e^2} = -\frac{1}{3e^4}(x - 2e^2)$

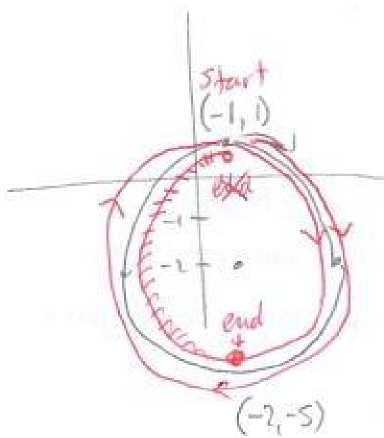
$y = -\frac{1}{3e^4}x + \frac{2e^2}{3e^4} + \frac{2}{e^2} = -\frac{2}{3e^4} + \frac{2}{3e^2} + \frac{2}{e^2}$

$y = -\frac{1}{3e^4} + \frac{8}{3e^2}$

4. (12pts) A particle moves along the path with parametric equations $x(t) = 1 + 3\sin t$, $y(t) = -2 + 3\cos t$ for $0 \le t \le 3\pi$. Sketch the path of motion and then describe the motion of the particle.

$x = 3\sin t$
$y = 3\cos t$ is a circle of radius 3 center $(0,0)$

$x = 1 + 3\sin t$
$y = -2 + 3\sin t$
circle of radius 3 center $(1,-2)$



When $t>0$ is small, $\sin t > 0$ so $1 + 3\sin t > 1$ so particle moves clockwise. For t in $[0, 3\pi]$ it goes around $1\frac{1}{2}$ times

5. (8pts) Identify the curve given in polar coordinates by $r = 2\cos\theta + 2\sin\theta$ by converting the equation to cartesian coordinates.

$r = 2\cos\theta + 2\sin\theta \quad | \cdot r$

$r^2 = 2r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = 2x + 2y$

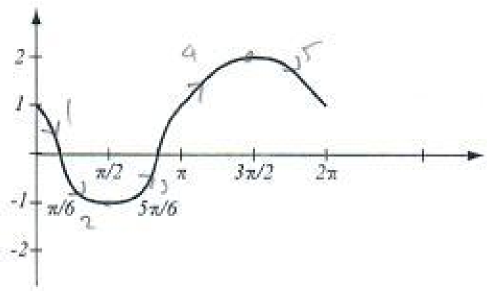
$x^2 - 2x + y^2 - 2y = 0 \quad | +1 +1$

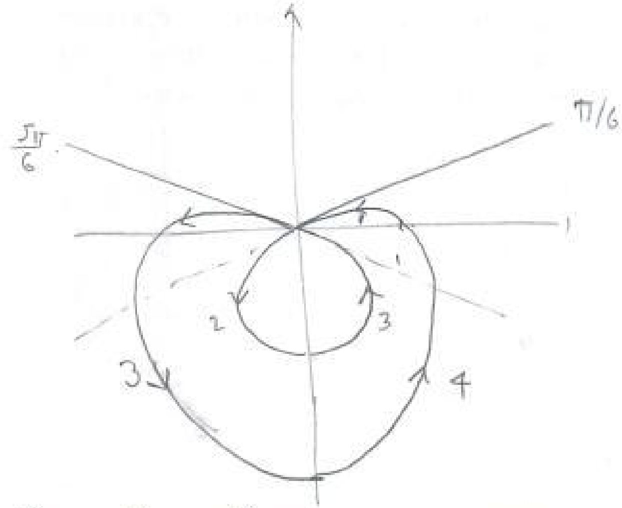
$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$

$(x-1)^2 + (y-1)^2 = 2$

circle, center $(1,1)$, radius $\sqrt{2}$

6. (12pts) The graph of $r = f(\theta)$ is given in cartesian coordinates. Use its intervals of increase and decrease to help you sketch the polar curve $r = f(\theta)$. Indicate which piece of the cartesian graph corresponds to which piece of the polar graph.





7. (14pts) The parametric curve $x = \frac{t^2}{2}$, $y = \frac{1}{9}(6t+9)^{\frac{3}{2}}$, $-1 \le t \le 3$ is given. Use an integral to find the length of the curve.

$$\ell = \int_{-1}^{3} \sqrt{t^2 + (6t+9)^{\frac{1}{2}}}\, dt = \int_{-1}^{3} \sqrt{\underbrace{t^2+6t+9}_{(t+3)^2}}\, dt = \int_{-1}^{3} t+3\, dt$$

$$x' = \frac{2t}{2} = t$$

$$y' = \frac{1}{9} \cdot \frac{3}{2} (6t+9)^{\frac{1}{2}} \cdot 6 = (6t+9)^{\frac{1}{2}}$$

$$= \frac{t^2}{2} \Big|_{-1}^{3} + 3(3-(-1))$$

$$= \frac{1}{2}(9-1) + 12 = 16$$

8. (18pts) A parametric curve is given by $x(t) = t^3 - 3t$, $y(t) = -t^3 + 12t$.
a) Find the points on the curve where the tangent line is horizontal or vertical.
b) Where does the curve go as $t \to \infty$ and $t \to -\infty$? (That is, find $\lim_{t\to\pm\infty} x(t)$, $\lim_{t\to\pm\infty} y(t)$.)
c) Plot the points from a) on a coordinate system and use them, along with information from b), or from plotting additional points, to get a graph of the curve. Recall that the curve moves in only one of general directions ↗ ↖ ↙ ↘ between points from a).

a) $x'(t) = 3t^2 - 3$

$y'(t) = -3t^2 + 12$

Horizontal: $y' = 0$

$-3t^2 + 12 = 0$

$t^2 = 4$

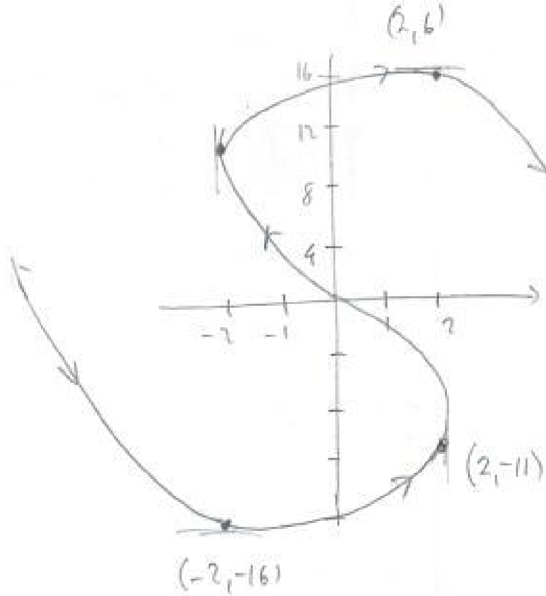
$t = \pm 2$

Vertical: $x' = 0$

$3t^2 - 3 = 0$

$t^2 = 1$

$t = \pm 1$



$\lim_{t\to\infty} t^3 - 3t = \lim_{t\to\infty} t^3\left(1 - \frac{3}{t^2}\right) =$

$t\to\infty$: $\infty(1-0) = \infty$

$t\to-\infty$: $-\infty(1-0) = -\infty$

$\lim_{t\to\infty} -t^3 + 12t = \lim_{t\to\infty} t^3\left(-1 + \frac{12}{t^2}\right)$

$t\to\infty$: $\infty(-1+0) = -\infty$

$t\to-\infty$: $-\infty(-1+0) = \infty$

t	x	y
-2	-2	-16
-1	2	-11
1	-2	11
2	2	16

Bonus. (10pts) The parametric curve $x = 2\cos^2 t$, $y = 2\sin t\cos t$, $0 \le t \le 2\pi$ is given. Eliminate the parameter to get the equation of the curve in cartesian coordinates. Then describe the how the curve is being traced by the parametrization. *(Hint: double-angle formulas for sine and cosine are useful.)*

$x = 2\cos^2 t = \cos(2t) + 1$

$y = 2\sin t \cos t = \sin(2t)$

$x - 1 = \cos(2t)$

$y = \sin(2t)$

$(x-1)^2 + y^2 = \cos^2(2t) + \sin^2(2t) = 1$

$(x-1)^2 + y^2 = 1$

circle of radius 1 centered at (1,0)

y goes from 0→1→-1→0 twice so goes around circle twice

Starting and ending at (2,0), goes around circle twice counterclockwise