Calculus 2 — Exam 2
MAT 308, Spring 2026 — D. Ivanšić

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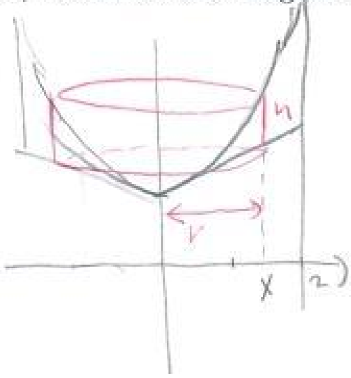
1. (24pts) The region bounded by the curves $y = e^x$, $y = x + 1$ and $x = 2$ is rotated around the y-axis. (Note that $y = e^x$ and $y = x + 1$ have only one common point.)
a) Sketch the solid and a typical cylindrical shell.
b) Set up the integral for the volume of the solid using the shell method.
c) Evaluate the integral.

a)



$r = x$

$h = e^x - (x+1)$

b) $V = \int_0^2 2\pi x \left(e^x - (x+1)\right) dx$

c) $= 2\pi \int_0^2 xe^x - x^2 - x \, dx = \begin{bmatrix} u = x & dv = e^x \\ du = 1dx & v = e^x \end{bmatrix}$

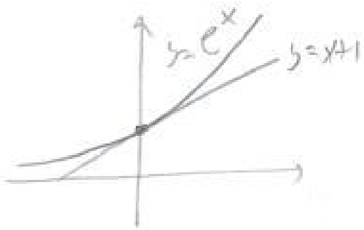
$= 2\pi \left(xe^x \Big|_0^2 - \int_0^2 e^x dx + \left(-\frac{x^3}{3} - \frac{x^2}{2} \right) \Big|_0^2 \right)$

$= 2\pi \left(2e^2 - 0e^0 - e^x \Big|_0^2 - \frac{1}{3}(2^3 - 0^3) - \frac{1}{2}(2^2 - 0) \right)$

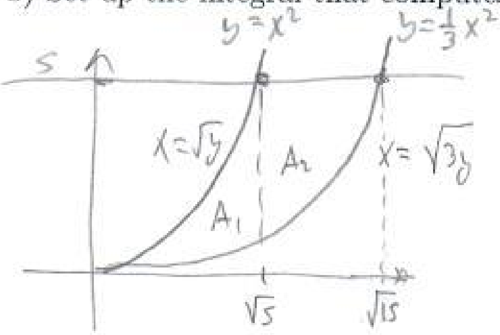
$= 2\pi \left(2e^2 - (e^2 - 1) - \frac{8}{3} - 2 \right)$

$= 2\pi \left(e^2 - 1 - \frac{8}{3} \right) = 2\pi \left(e^2 - \frac{11}{3} \right)$

Note, $x+1$ is tangent to e^x



2. (14pts) Consider the half of the region bounded by the curves $y = x^2$, $y = \frac{1}{3}x^2$ and $y = 5$ that is in the first quadrant.
a) Sketch the region.
b) Set up the integral that computes its area. Simplify, but do not evaluate the integral.



$A = \int_0^5 \sqrt{3y} - \sqrt{y} \, dy = (\sqrt{3} - 1) \int_0^5 \sqrt{y} \, dy$

Using x: $A = A_1 + A_2 = \int_0^{\sqrt{5}} x^2 - \frac{1}{3}x^2 dx + \int_{\sqrt{5}}^{\sqrt{15}} 5 - \frac{1}{3}x^2 dx$

$= \int_0^{\sqrt{5}} \frac{2}{3}x^2 dx + \int_{\sqrt{5}}^{\sqrt{15}} 5 - \frac{1}{3}x^2 dx$

$x^2 = 5$

$x = \pm\sqrt{5}$

$\frac{1}{3}x^2 = 5$

$x^2 = 15$

$x = \pm\sqrt{15}$

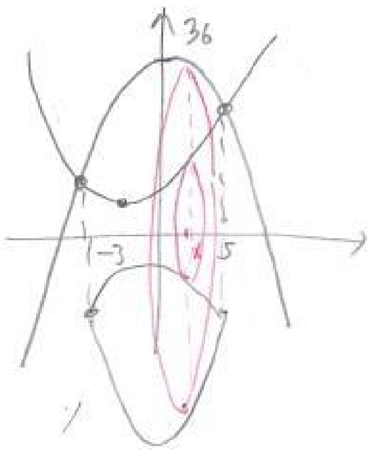
14

3. (~~16~~pts) Rotate the region bounded by the curves $y = x^2 + 4x + 6$ and $y = 36 - x^2$ about the x-axis to get a solid.

a) Sketch the solid and a typical cross-sectional washer.

b) Set up the integral for the volume of the solid.

Don't Simplify, but do not evaluate the integral.



vertex of x^2+4x+6:

$-\frac{b}{2a} = -\frac{4}{2\cdot 1} = -2$

$(-2)^2+4(-2)+6$

$= 4-8+6 = 2$

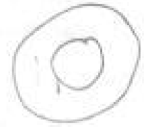
$x^2+4x+6 = 36-x^2$

$2x^2+4x-30=0$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5, 3$



$r_2 = 36-x^2, \quad r_1 = x^2+4x+6$

$$V = \int_{-5}^{3} \pi\left((36-x^2)^2 - (x^2+4x+6)^2\right)dx$$

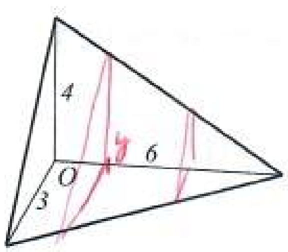
15

4. (~~16~~pts) At point O, the sides of the tetrahedron shown below all meet at right angles.

a) Devise a way to take cross-sections of this solid and draw two of them.

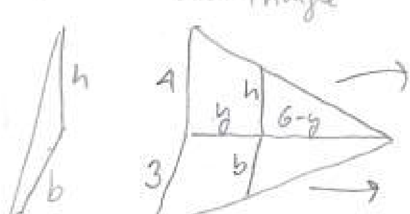
b) Determine the area of each cross-section as a function of a suitable variable. Similar triangles will be useful.

c) Set up the integral for the volume of the tetrahedron. Simplify, but do not evaluate the integral.



$$V = \int_0^6 A(y)\,dy = \int_0^6 \frac{1}{2} h(y)\, b(y)\, dy$$

back triangle



bottom triangle

$\frac{6-y}{6} = \frac{h}{4}, \quad h = \frac{4}{6}(6-y) = \frac{2}{3}(6-y)$

$\frac{6-y}{6} = \frac{b}{3} \quad b = \frac{3}{6}(6-y) = \frac{1}{2}(6-y)$

$$V = \int_0^6 \frac{1}{2}\,\frac{1}{2}(6-y)\,\frac{2}{3}(6-y)\,dy = \int_0^6 \frac{1}{6}(6-y)^2\,dy$$

16

5. (~~14~~pts) Find the length of the curve $y = \ln \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$.

$$L = \int_{\pi/4}^{\pi/3} \sqrt{1 + (\ln \cos x)'^2}\, dx = \int_{\pi/4}^{\pi/3} \sqrt{1 + \left(\frac{1}{\cos x}(-\sin x)\right)^2}\, dx$$

$$= \int_{\pi/4}^{\pi/3} \underbrace{\sqrt{1 + \tan^2 x}}_{\sec^2 x}\, dx = \int_{\pi/4}^{\pi/3} \sec x \, dx = \ln|\sec x + \tan x|\Big|_{\pi/4}^{\pi/3}$$

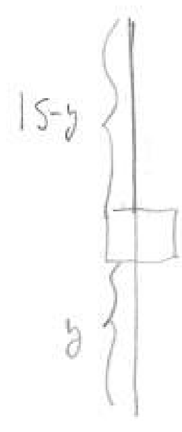
$$= \ln\left(\sec\tfrac{\pi}{3} + \tan\tfrac{\pi}{3}\right) - \ln\left|\sec\tfrac{\pi}{4} + \tan\tfrac{\pi}{4}\right)$$

$$= \ln|2 + \sqrt{3}| - \ln|\sqrt{2} + 1|$$

14

6. (~~16~~pts) Set up the integral to find the work expended when five people take the elevator from the first to the sixth floor of Faculty Hall. Assume $g = 10$, the empty elevator weighs 150kg, the five people together weigh 320kg, each floor has height 3m and the cable lifting the elevator weighs 4kg per meter. Simplify, but do not evaluate the integral.

from 1st to 6th fl



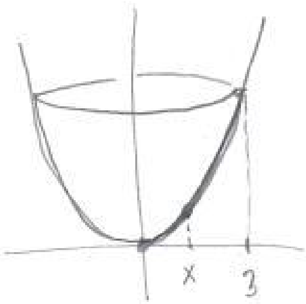
5 floors $= 5 \cdot 3 = 15$ height

mass $= 150 + 320 + 4(15 - y) = 530 - 4y$

$$W = \int_0^{15} F(y)\, dy = \int_0^{15} (530 - 4y)10\, dy$$

$$= \int_0^{15} 5300 - 40y \, dy$$

Bonus (10pts) If we rotate the curve $y = x^2$ around the y-axis, we get a surface called a *paraboloid*. Compute the surface area of the part of the paraboloid that comes from rotating the section of $y = x^2$ for $0 \leq x \leq 3$.



$$S = \int_0^3 2\pi r \, ds = \int_0^3 2\pi x \sqrt{1+((x^2)')^2}\, dx$$

$$= \int_0^3 2\pi x \sqrt{1+4x^2}\, dx = \left[\begin{array}{ll} u = 1+4x^2 & x=3,\ u=37 \\ du = 8x\,dx & x=0,\ u=1 \\ \frac{1}{8}du = x\,dx & \end{array}\right]$$

$$= \int_1^{37} 2\pi \sqrt{u} \cdot \frac{1}{8}\, du$$

$$= \frac{1}{4}\pi \cdot \frac{2}{3} u^{3/2} \Big|_1^{37} = \frac{\pi}{6}\left(37^{\frac{3}{2}} - 1\right)$$