Calculus 2 — Exam 0
MAT 308, Spring 2026 — D. Ivanšić

Name: Saul Ocean

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\frac{d}{dx}\left(5x^9-\pi^3-3\sqrt[6]{x^{11}}+\frac{5}{x^2}\right)= 5\cdot 9x^8-0-3\cdot\frac{11}{6}x^{\frac{5}{6}}+5(-2)x^{-3}$

$x^{\frac{11}{6}}$ $\quad 5x^{-2}$

$= 45x^8-\frac{11}{2}x^{\frac{5}{6}}-\frac{10}{x^3}$

2. (6pts) $\frac{d}{dx}\left(\sqrt{x}e^{3x+2}\right)= \frac{1}{2\sqrt{x}}e^{3x+2}+\sqrt{x}\,e^{3x+2}\cdot 3$

$= e^{3x+2}\left(\frac{1}{2\sqrt{x}}+3\sqrt{x}\right)=e^{3x+2}\,\frac{1+6x}{2\sqrt{x}}$

3. (8pts) $\frac{d}{du}\frac{5u^4}{(u^2+2u+1)^3}= \frac{20u^3(u^2+2u+1)^3-5u^4 3(u^2+2u+1)^2\overset{2(u+1)}{(2u+2)}}{(u^2+2u+1)^6}$

$2u^2+4u+2-3u^2-\cancel{3u}$

$= \frac{10u^3\cancel{(u^2+2u+1)^2}\left(2(u^2+2u+1)-3u(u+1)\right)}{(u^2+2u+1)^{\cancel{6}\,4}} = \frac{10u^3(-u^2+u+2)}{(u^2+2u+1)^4}$

4. (4pts) $\frac{d}{dx}\frac{1}{(\cos x-\sin x)^2}= \frac{d}{dx}(\cos x-\sin x)^{-2}=-2(\cos x-\sin x)^{-3}(-\sin x-\cos x)$

$= \frac{2(\sin x+\cos x)}{(\cos x-\sin x)^3}$

5. (6pts) $\frac{d}{dx}\arctan(\ln(x^3-2x))= \frac{1}{1+(\ln(x^3-2x))^2}\cdot\frac{1}{x^3-2x}(3x^2-2)$

$= \frac{3x^2-2}{\left(1+(\ln(x^3-2x))^2\right)(x^3-2x)}$

6. (7pts) Find the first and second derivatives of $f(x)=\tan(\sqrt{x})$.

$y=\tan\sqrt{x}$

$y'=\sec^2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}=\frac{\sec^2\sqrt{x}}{2\sqrt{x}}$

$y''=\frac{1}{2}\,\frac{2\sec\sqrt{x}\cdot\sec\sqrt{x}\tan\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\cdot\sqrt{x}-\sec^2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}}{\sqrt{x}^{\,2}}=\cdot\frac{2\sqrt{x}}{2\sqrt{x}}=\frac{2\sqrt{x}\sec^2\sqrt{x}\tan\sqrt{x}-\sec^2\sqrt{x}}{4x\sqrt{x}}$

$=\frac{\sec^2\sqrt{x}\left(2\sqrt{x}\tan\sqrt{x}-1\right)}{4x\sqrt{x}}$

7. (5pts) Let $f(x) = x \ln x$. Take the first four derivatives of f, and try to spot the pattern. What is $f^{(29)}(x)$, the 29th derivative of f? How about $f^{(n)}(x)$?

$y = x \ln x$

$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$y'' = \frac{1}{x} = x^{-1}$

$y''' = -x^{-2}$

$y^{(4)} = (-1)(-2)x^{-3}$

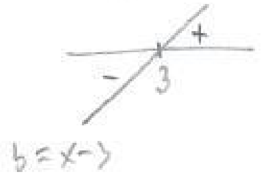
$y^{(5)} = \underbrace{(-1)(-2)(-3)}_{\uparrow\ \text{2 less than derivative}} x^{-4}$ ← one less than derivative

$y^{(29)} = (-1)(-2)\dots(-27)x^{-28} = (-1)^{27}\,27!\,x^{-28}$

$y^{(n)} = (-1)(-2)\dots(-(n-2))\,x^{-(n-1)} = (-1)^{n-2}(n-2)!\,x^{-(n-1)}$

Find the following limits. Use L'Hospital's rule if needed.

8. (4pts) $\lim\limits_{x\to 3+} \dfrac{1}{x-3} = \dfrac{1}{0+} = \infty$



$x > 3$ or: $x - 3 > 0$ or $3.01 - 3 = 0.01 > 0$

9. (5pts) $\lim\limits_{x\to\infty} \dfrac{x^2 - 6x + 1}{3x^4 + x^2} = \lim\limits_{x\to\infty} \dfrac{x^2\left(1 - \overbrace{\frac{6}{x}}^{\to 0} + \overbrace{\frac{1}{x^2}}^{\to 0}\right)}{x^4\left(3 + \underbrace{\frac{1}{x^2}}_{\to 0}\right)} = \lim\limits_{x\to\infty} \dfrac{1}{x^2} \cdot \dfrac{1-0+0}{3+0}$

$= 0 \cdot \frac{1}{3} = 0$

10. (7pts) $\lim\limits_{x\to 0} x^4 \ln|x| = \left[\begin{matrix} 0 \cdot \ln 0 = 0 \cdot (-\infty) \\ \text{indet. form} \end{matrix}\right] = \lim\limits_{x\to 0} \dfrac{\ln|x|}{\frac{1}{x^4}} = \left[\dfrac{-\infty}{\frac{1}{0^+}} = \dfrac{-\infty}{\infty} = \text{indet. form}\right]$

$\overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{x}}{-4x^{-5}} = \lim\limits_{x\to 0} \dfrac{\overset{x^{-1-(-5)}}{x^4}}{-4} = \dfrac{0}{-4} = 0$

Find the following antiderivatives.

11. (7pts) $\int 5x^5 + \frac{3}{\sqrt{1-x^2}} + \sqrt[4]{x^9} + a^3\,dx =$ $5\cdot\frac{x^6}{6} + 3\arcsin x + \frac{x^{\frac{13}{4}}}{\frac{13}{4}} + a^3x$

$x^{\frac{9}{4}}$

$= \frac{5x^6}{6} + 3\arcsin x + \frac{4}{13}x^{\frac{13}{4}} + a^3x + C$

12. (3pts) $\int \tan\left(2x+\frac{\pi}{3}\right)\,dx =$ $\frac{\ln\left|\sec\left(2x+\frac{\pi}{3}\right)\right|}{2} + C$

13. (7pts) $\int x^6(\sqrt{x} + x\sqrt[3]{x})\,dx =$ $\int x^6\left(x^{\frac12} + x\cdot x^{\frac13}\right)dx = \int x^6\left(x^{\frac12} + x^{\frac43}\right)dx$

$= \int x^{\frac{13}{2}} + x^{\frac{22}{3}}\,dx = \frac{2}{15}x^{\frac{15}{2}} + \frac{3}{25}x^{\frac{25}{3}} + C$

Use the substitution rule in the following integrals:

14. (7pts) $\int \boxed{\sec^2 x}\,\sqrt[5]{\tan x}\,\boxed{dx} =$ $\left[\begin{array}{l} u = \tan x \\ du = \boxed{\sec^2 x\,dx} \end{array}\right] = \int \sqrt[5]{u}\,du$

$u^{1/5}$

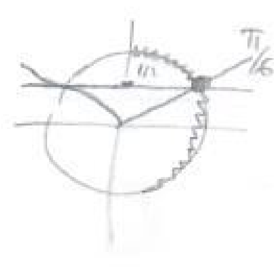
$= \frac{5}{6}u^{\frac65} = \frac{5}{6}\tan^{\frac65}x + C$

15. (10pts) $\int_{\sqrt{\frac32}}^{\sqrt{3}} \frac{\boxed{x\,dx}}{\sqrt{1-\left(\frac{x^2}{3}\right)^2}} =$ $\left[\begin{array}{ll} u = \frac{x^2}{3} & x=\sqrt{3},\ u=1 \\ du = \frac{2x}{3}\,dx & x=\sqrt{\frac32},\ u=\frac{\frac32}{3}=\frac12 \\ \frac32\,du = \boxed{x\,dx} & \end{array}\right]$

$= \int_{1/2}^{1} \frac{1}{\sqrt{1-u^2}}\cdot\frac32\,du = \frac32\arcsin u\Big|_{1/2}^{1} = \frac32\left(\arcsin 1 - \arcsin\frac12\right)$

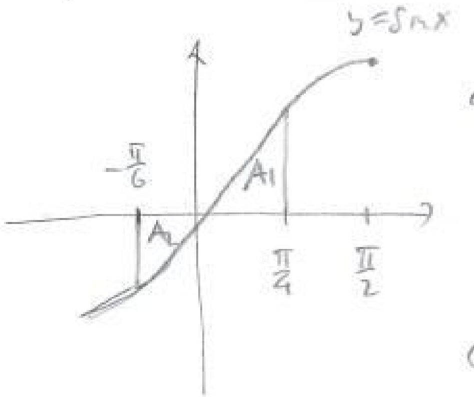
$= \frac32\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac32\cdot\frac{\pi}{3} = \frac{\pi}{2}$

$\sin\theta = \frac12$



16. (8pts) Consider the integral $\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x\, dx$.

a) Draw a picture and use the "area" interpretation to explain the what the integral represents.
b) Use the picture to estimate whether the integral is positive or negative.
c) Evaluate the integral to verify your finding in b).



a) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x\, dx = -A_2 + A_1$

b) $A_1 - A_2 > 0$ since it looks like $A_1 > A_2$

c) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x\, dx = -\cos x \Big|_{-\pi/6}^{\pi/4} = -\left(\cos\frac{\pi}{4} - \cos\left(-\frac{\pi}{6}\right)\right)$

$= -\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}-\sqrt{3}}{2} = \frac{\sqrt{3}-\sqrt{2}}{2} > 0$

Bonus. (10pts) The rear inside cover of our book claims that

$$\int \frac{\sqrt{x^2-a^2}}{x^2}\, dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x + \sqrt{x^2-a^2}\right| + C$$

Verify this formula by differentiating.

$$\frac{d}{dx}\left(-\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right|\right) = -\frac{\frac{1}{2\sqrt{x^2-a^2}}\cdot 2x\cdot x - \sqrt{x^2-a^2}\cdot 1}{x^2} + \frac{1}{x+\sqrt{x^2-a^2}}\left(1 + \frac{1}{\cancel{2}\sqrt{x^2-a^2}}\cancel{2}x\right)$$

$$= -\frac{\frac{x^2}{\sqrt{x^2-a^2}} - \sqrt{x^2-a^2}}{x^2}\cdot\frac{\sqrt{x^2-a^2}}{\sqrt{x^2-a^2}} + \frac{1}{\cancel{x+\sqrt{x^2-a^2}}}\cdot\frac{\cancel{\sqrt{x^2-a^2}+x}}{\sqrt{x^2-a^2}} \qquad \sqrt{x^2-a^2}^{\,2}$$

$$= -\frac{x^2-(x^2-a^2)}{x^2\sqrt{x^2-a^2}} + \frac{1}{\sqrt{x^2-a^2}} = -\frac{a^2}{x^2\sqrt{x^2-a^2}} + \frac{x^2}{x^2\sqrt{x^2-a^2}} = \frac{x^2-a^2}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{x^2}$$