

College Algebra — Joysheet 1  
MAT 140, Spring 2026 — D. Ivanšić

Name: \_\_\_\_\_

Covers: *JIT* <sup>6, 7, 11, 12,</sup><sub>14, 15, 17</sub> Show all your work!

Write the sets in interval notation and sketch them on the number line.

1. (3pts)  $\{x \mid x \leq 4\}$

2. (3pts)  $\{x \mid -2 < x < 0\}$

Solve the equations.

3. (3pts)  $2x - 4 = 7x - 2$

4. (4pts)  $5 + 2(a - 3) = -3(a + 3)$

Simplify and write in standard form:

5. (4pts)  $(x^2 - 5x + 2)(x - 3) =$

6. (4pts)  $x^2(x + 4) - (x - 7)^2 =$

Simplify and write the answer so all exponents are positive:

7. (2pts)  $y^6(3y)^4 =$

8. (2pts)  $\frac{(2x)^6}{x^4} =$

9. (3pts)  $(u^5v^{-2})^4u^{-7}v^7 =$

10. (5pts)  $(4a^{-2}b^4)^3(a^5c^{-2})^{-4} =$

11. (7pts)  $\frac{(6x^5y^{-3})^3}{(2x^{-2}y^2)^4} =$

Factor the following.

12. (4pts)  $x^2 + 3x - 10 =$

13. (4pts)  $x^2 - 10x + 24 =$

Use the *ac*-method or another method to factor. Show how you got your answer.

14. (6pts)  $2x^2 + 7x - 15 =$

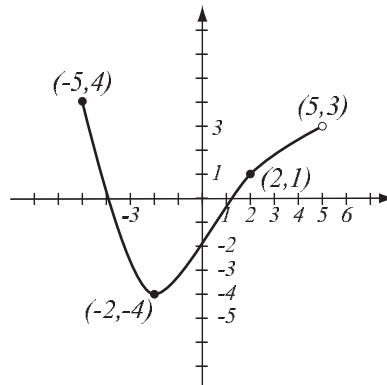
15. (6pts)  $4x^2 - 8x - 5 =$

1. (11pts) Draw the triangle with vertices  $A = (-4, 0)$ ,  $B = (5, 3)$  and  $C = (6, 0)$ .
- Compute the lengths of all sides of the triangle.
  - Determine algebraically if the triangle is right.
  - Find the area of the triangle.

2. (10pts) Find the equation of the circle if the endpoints of its diameter are  $(-2, 3)$  and  $(4, 0)$ . Draw the circle.

3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- Find  $f(-2)$  and  $f(6)$ .
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = 1$ ?



4. (12pts) The function  $f(x) = x^4 - 6x^3 + 2x^2 - 5$  is given.

a) Use your calculator to accurately graph its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (9pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt{x}}{2x - 7}$$

$$g(x) = \frac{4}{x^2 - 8x}$$

6. (10pts) Let  $f(x) = x^3 - 4x$ . Find the following (simplify where appropriate).

$$h(4) =$$

$$h\left(\frac{1}{2}\right) =$$

$$h(-2t) =$$

$$h(t + 3) =$$

1. (5pts) Find the equation of the line (in form  $y = mx + b$ ) whose  $x$ -intercept is 3 and passes through point  $(-1, 2)$ . Draw the line.

2. (10pts) Find the equation of the line (in form  $y = mx + b$ ) passing through  $(2, -4)$  that is perpendicular to the line  $3x - 2y = 6$ . Draw both lines.

3. (9pts) Draw the quadrangle with vertices  $A = (-1, -3)$ ,  $B = (4, 1)$ ,  $C = (-1, 7)$  and  $D = (-6, 3)$ .

a) Find the slopes of the sides of the quadrangle.

b) Use slopes to check if the quadrangle is a parallelogram (has two pairs of parallel sides).

c) Is it a rectangle (a parallelogram with adjacent sides perpendicular)? Use slopes to check.

4. (4pts) The median home price in the US at end of 2018 was \$322,800. At end of 2024, it was \$419,300. What is the average rate of change of median home price from 2018 to 2024? What are the units for the average rate of change?

5. (12pts) The water bill for a household was \$52.03 in a month when it used 42 hundred-gallons of water. In another month, it used 51 hundred-gallons and was billed \$56.44.
- Assuming that water cost  $C(x)$  is a linear function of the amount of water  $x$  used (in hundred-gallons), write a formula for  $C(x)$ .
  - What is the cost if no water is used during a month? What is the meaning of this number?
  - What is the meaning of the slope in this example?

6. (20pts) A statistician is trying to establish a trend for the population of Japan. In the table,  $P$  is the population, in millions, of Japan in year  $x$ . Solve the problems below with accuracy 6 decimal points.
- Draw the scatterplot of the data. Does the relationship look linear?
  - Use two points in the scatterplot to get an equation of a line that models the relationship between  $P$  and  $x$ . Draw the line on the graph.
  - Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.
  - Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $x$  and  $P$ ?
  - What population of Japan can we expect for 2030?

$x$	$P$
2000	126.9
2005	127.8
2010	128.1
2015	127.1
2020	126.1
2025	123.2

Solve the inequalities. Write your solution in interval notation.

1. (5pts)  $-6 < 4x - 1 < 7$

2. (7pts)  $2x + 3 < 7$  or  $3x + 4 > 21$

3. (6pts) Find the domain of the function in interval notation:  $f(x) = \frac{\sqrt{5 - 8x}}{x + 4}$ .

4. (14pts) Boat operators Bugsy's and Daffy's charge the following rates for an excursion on a lake. Bugsy's charges \$50 plus \$24 per person. Daffy's charges \$350, which includes 10 people, plus \$19 per person for every person over the first ten. Assuming your group has at least 10 people, for which number of people is Bugsy's the better option? Solve as an inequality.

**5.** (14pts) Kristi lives in Benton, and Laura in Madisonville. One day, leaving at the same time, Kristi drives on the highway toward Madisonville at 63 mph while Laura drives at 57 mph toward Benton on a scenic road that is 12 miles longer than the highway. Laura arrives in Benton 20 minutes after Kristi arrives in Madisonville.

- a) How long did it take for each of them to drive to their destinations?
- b) How far is Madisonville from Benton along the highway?

**6.** (14pts) You need 35 cans of soda for a party. A can of lemon soda costs 79 cents and a can of cola costs 91 cents. After you buy some lemon soda cans and some cola cans (35 altogether), you realize that, if you reversed the amounts of cans of each type you bought, it would have cost you 36 cents extra. How many cans of each soda type did you buy?

1. (10pts) Use your calculator to accurately sketch the graph of the function

$f(x) = \frac{x^2 + 9}{x}$ . (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = \frac{x}{x^2 - 4}$ ,  $g(x) = \sqrt{4x + 13}$ . Find the following (simplify where possible):

$$(f + g)(3) =$$

$$(fg)(7) =$$

$$\frac{f}{g}(x) =$$

$$(g \circ f)(4) =$$

$$(f \circ g)(x) =$$

The domain of  $(f - g)(x)$  in interval notation

3. (8pts) Consider the function  $h(x) = \frac{5}{\sqrt{x-7}}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

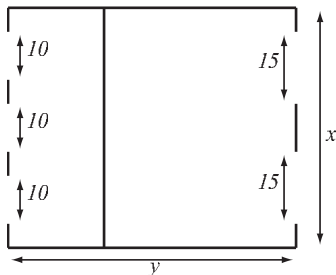
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} \frac{1}{2}x + 3, & \text{if } -2 \leq x < 4 \\ 2x - 10 & \text{if } x \geq 4 \end{cases}$$

5. (14pts) Fulton University needs a garage for its maintenance vehicles with area 3000 square feet, divided into two sections, as in the picture. The sections have doors of width either 10 or 15 feet. The university wishes to minimize the construction cost, which is same as minimizing the total length of the walls.

a) Express the total length of the walls as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the garage for which the total length of the walls is minimal? What is the minimal wall length?



1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^5 - 4x^3 + 5x$$

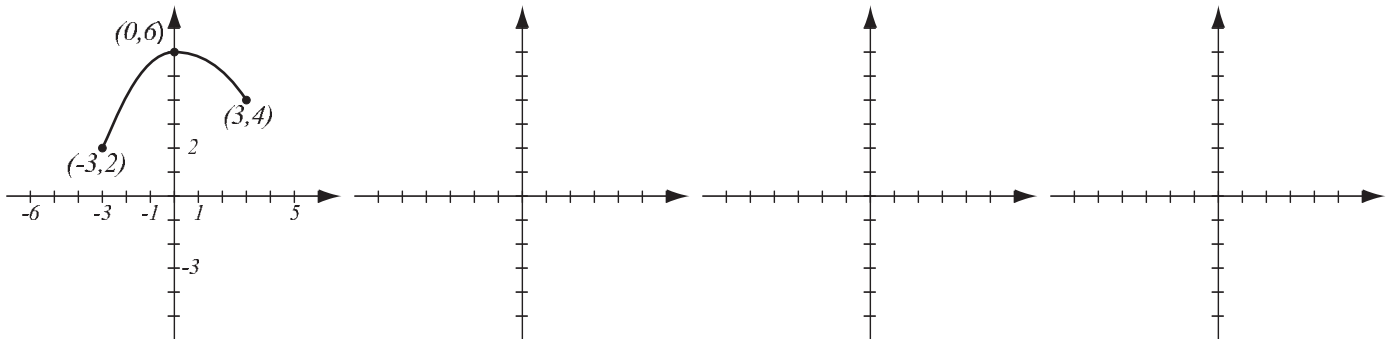
$$g(x) = x^4 - 3x^2 + 7$$

$$h(x) = x^3 + 3x^2 - 4x$$

2. (16pts) Draw the graphs of  $f(x) = -(x+3)^2$  and  $g(x) = -3 + 2\sqrt{-x}$  using transformations. Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of  $y = |x|$ , shifted right 4 units.
  - shape of  $y = x^3$ , stretched horizontally by factor 3, then reflected in  $x$ -axis
  - shape of  $y = \frac{1}{x}$ , reflected about the  $y$ -axis, then shifted left 3 units, then stretched vertically by factor 4.

4. (13pts) The graph of  $f(x)$  is drawn below. On separate coordinate systems, sketch the graphs of the functions  $f(3x)$ ,  $-f(x+2)$  and  $\frac{1}{2}f(-x)$  and label all the relevant points.



College Algebra — Joysheet 7  
MAT 140, Spring 2026 — D. Ivanšić

Name: \_\_\_\_\_

Covers: JIT <sup>13, 21, 22,</sup><sub>23, 25</sub> Show all your work!

Use formulas to expand:

1. (4pts)  $(3a + 2b)^2 =$

2. (5pts)  $(u^2 - 7v^3)^2 =$

3. (4pts)  $(2x - y^2)(2x + y^2) =$

4. (8pts) Compute expressions with fractions by hand.

$$\frac{4}{15} \cdot \frac{35}{6} =$$

$$\frac{14}{27} \div \frac{49}{18} =$$

$$\frac{7}{6} + \frac{11}{18} =$$

$$\frac{15}{28} - \frac{20}{35} =$$

Multiply or divide the rational expressions.

5. (7pts)  $\frac{x - 4}{3x + 21} \cdot \frac{6x^2 + 42x}{x^2 + x - 20} =$

6. (7pts)  $\frac{3x - 5}{3x^2 + 14x - 5} \div \frac{3x + 1}{9x^2 - 1} =$

Add or subtract the rational expressions.

7. (6pts)  $\frac{x + 3}{x + 7} - \frac{x^2 - 7}{x^2 + 5x - 14} =$

8. (8pts)  $\frac{-2x + 8}{2x^2 - 13x + 21} + \frac{x + 15}{x^2 + 3x - 18} =$

Simplify the following expressions, assuming all variables are positive.

9. (3pts)  $\sqrt[3]{32u^6v^4} =$

10. (4pts)  $\sqrt{6x^3y^4} \sqrt{30xy} =$

11. (4pts)  $(1 + 3\sqrt{5})(3\sqrt{20} - 4) =$

College Algebra — Joysheet 8  
MAT 140, Spring 2026 — D. Ivanšić

Name: \_\_\_\_\_

*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $(1 + 2i)^3 =$

2. (6pts)  $\frac{7 + i}{3 - 5i} =$

3. (4pts) Simplify and justify your answer.

$i^{163} =$

4. (8pts) The number of smartphones in a warehouse (in thousands) is described by the function  $T(x) = x^2 - 14x + 70$ , where  $x$  is the number of days after March 25th.

a) On what dates were there 37 thousand phones in the warehouse?

b) On what date did the number of smartphones in the warehouse reach its minimum?

5. (8pts) Solve the equation:  $x^4 + 2x^2 - 63 = 0$

6. (6pts) Solve by completing the square.

$x^2 + 10x + 30 = 0$

**7.** (12pts) The quadratic function  $f(x) = -4x^2 + 4x + 15$  is given. Do the following without using the calculator.

a) Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

**8.** (12pts) The base of an isosceles triangle is 14 cm (neither of the two equal sides is the base). If the height on that base is 5 cm shorter than the two equal sides of the triangle, what is the length of the equal sides?

College Algebra — Joysheet 9  
MAT 140, Spring 2026 — D. Ivanšić

Name: \_\_\_\_\_

*Covers: 3.3, 3.4, 3.5 Show all your work!*

1. (4pts) Solve the equation.

$$|3x + 4| = 8$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x + 2| < 7$$

$$|4x - 5| \geq 1$$

Solve the equations:

3. (8pts)  $\frac{x}{x+3} - \frac{2x-3}{x-5} = \frac{3x+33}{x^2-2x-15}$

4. (8pts)  $x = 2 - \sqrt{16 - 8x}$

5. (14pts) A model rocket is launched. When its motor cuts off, it is at height 220 feet heading upwards with velocity 180 feet per second. Its height in feet after  $t$  seconds is given by  $s(t) = -16t^2 + 180t + 220$ .

a) Sketch the graph of the height function.

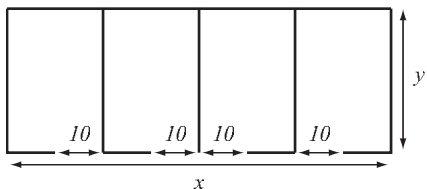
b) When does the rocket reach its greatest height, and what is that height?

c) When does the rocket fall to the ground?

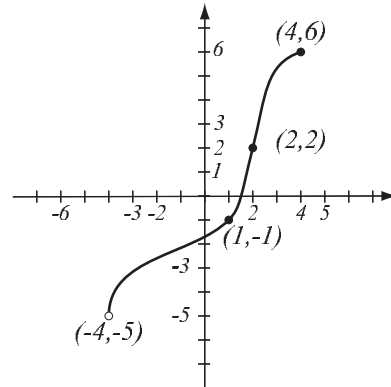
6. (14pts) Caroline is planning a building to house two quick-serve restaurants, each with 10-by-10 entrances at the corner (see picture). She has budgeted for total wall length 500 feet and her goal is to maximize the total enclosed area.

a) Express the area of the building as a function of one of the sides of the rectangle. What is the domain of this function?

c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the greatest total area and what is the greatest area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points and considering the asymptote.



2. (12pts) Let  $f(x) = \frac{1+x}{2+x}$ . Find the formula for  $f^{-1}$ . Find the ranges of  $f$  and  $f^{-1}$ .

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_2 64 = \quad \log_3 \frac{1}{27} = \quad \log_5 \sqrt[4]{125} = \quad \log_{\sqrt{b}} b^4 =$$

4. (4pts) Use the change-of-base formula and your calculator to find  $\log_4 24$  with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$2,250 and annual interest rate of 3.15%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	$n$	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of  $f(x) = \ln(2x - 9)$ . Write your solution in interval notation.

7. (8pts) In an account with principal \$1,000, the amount  $A$  after 2 years depends on the interest  $r$ , compounded annually, according to the formula  $A = 1000(1 + r)^2$ .

- Determine the amount in the account if the interest rate is 3% or 4%.
- Find a formula for the inverse function and explain what it represents.
- What should the interest rate be if we wish to have \$1,080 or \$1,120 in the account in two years?

8. (7pts) Using transformations, draw the graph of  $f(x) = -\log_2(-x)$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If  $\log_a 2 = 0.3562$  and  $\log_a 5 = 0.8271$ , calculate:

$$\log_a 10 =$$

$$\log_a \frac{4}{25} =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\ln(\sqrt{e} x^2 y^3) =$$

$$\log_4 \frac{64x^5 \sqrt[3]{y^2}}{x^3 z^4} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$3 \log_5(2u^4) - \log_5(v^2) - 2 \log_5(6u^2) =$$

$$3 \log_2(x^2 - 3x - 28) - \log_2(x - 7) + 2 \log_2(x + 4) =$$

4. (2pts) Simplify.  $\log 10^{\sqrt{x}+2} =$   $6^{\log_6(17-2u)} =$

Solve the equations.

5. (5pts)  $4^{3x-1} = \frac{1}{2^{x+7}}$

6. (6pts)  $3^{2x-1} = e^{4-2x}$

7. (7pts)  $e^{2x} + 4e^x - 12 = 0$

8. (12pts) Weederton had a population of 12,000 in 2015 and 19,000 in 2025. Assume the population of Weederton grows exponentially.

a) Write the function describing the number  $P(t)$  of people in Weederton  $t$  years after 2015. Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 40,000?