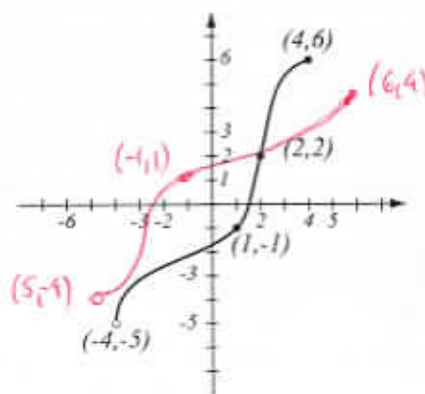


1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points and considering the asymptote.



- a) Yes - passes horizontal line test
b) see picture

2. (12pts) Let $f(x) = \frac{1+x}{2+x}$. Find the formula for f^{-1} . Find the ranges of f and f^{-1} .

$$y = \frac{1+x}{2+x}$$

$$f^{-1}(y) = \frac{1-2y}{y-1}$$

$$\text{Range } f = \text{Domain } f^{-1} = (-\infty, 1) \cup (1, \infty)$$

can't have $y-1=0$
 $y=1$

$$y(2+x) = 1+x$$

$$2y + yx = 1+x$$

$$yx - x = 1 - 2y$$

$$x(y-1) = 1-2y$$

$$x = \frac{1-2y}{y-1}$$

$$\text{Range } f^{-1} = \text{Domain } f = (-\infty, -2) \cup (-2, \infty)$$

can't have $2+x=0$
 $x=-2$

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_2 64 = 6$$

$$2^? = 64$$

$$\log_3 \frac{1}{27} = -3$$

$$3^? = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$$

$$\log_5 \sqrt[4]{125} = \frac{3}{4}$$

$$5^? = \sqrt[4]{125} = \sqrt[4]{5^3} = 5^{\frac{3}{4}}$$

$$\log_{\sqrt{b}} b^4 = 8$$

$$(\sqrt{b})^? = b^4$$

$$(\sqrt{b})^2 = b$$

$$((\sqrt{b})^2)^4 = b^4$$

$$\sqrt{b^8} = b^4$$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_4 24$ with accuracy 6 decimal places. Show how you obtained your number.

$$\log_4 24 = \frac{\ln 24}{\ln 4} = 2.292481$$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$2,250 and annual interest rate of 3.15%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- a) Write the general formula for the amount, replacing the variables by numbers, if known.
 b) Use the table feature on your calculator to quickly compute amounts after 1 year.
 c) Does compounding more often make a big difference?

| Frequency: every | n | Amount after 1 year |
|------------------|--|---------------------|
| year | 1 | 2320.88 |
| quarter | 4 | 2321.72 |
| month | 12 | 2321.91 |
| day | 365 | 2322.00 |
| hour | $365 \cdot 24 = 8760$ | 2322.00 |
| second | $365 \cdot 24 \cdot 3600 = 31,536,000$ | 2322.01 |

$$A = 2250 \left(1 + \frac{0.0315}{n}\right)^n$$

c) Does not make a very big difference in few cases, especially when we compound very, very often

6. (3pts) Find the domain of $f(x) = \ln(2x - 9)$. Write your solution in interval notation.

Must have $2x - 9 > 0$
 $2x > 9$
 $x > \frac{9}{2}$
 $\left[\frac{9}{2}, \infty\right)$

7. (8pts) In an account with principal \$1,000, the amount A after 2 years depends on the interest r , compounded annually, according to the formula $A = 1000(1+r)^2$.

- a) Determine the amount in the account if the interest rate is 3% or 4%.
 b) Find a formula for the inverse function and explain what it represents.
 c) What should the interest rate be if we wish to have \$1,080 or \$1,120 in the account in two years?

a) $A = 1000(1+r)^2$

| r | $1000(1+r)^2$ |
|------|---------------|
| 0.03 | 1060.90 |
| 0.04 | 1081.60 |

Given amount A , this tells us interest rate needed to have A dollars after 2 years

b) $A = 1000(1+r)^2$

$$\frac{A}{1000} = (1+r)^2$$

$$\sqrt{\frac{A}{1000}} = 1+r$$

$$r = \sqrt{\frac{A}{1000}} - 1$$

c) $r = \sqrt{\frac{A}{1000}} - 1$

| | |
|------|----------------------------|
| 1080 | $0.0392305 \approx 3.92\%$ |
| 1120 | $0.0583005 \approx 5.83\%$ |

8. (7pts) Using transformations, draw the graph of $f(x) = -\log_2(-x)$. Explain how you transform the graph of a basic function in order to get the graph of f . Show at least one point on the graph, and asymptotes to the graph, if any.

