

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $(1 + 2i)^3 = (1 + 2i)^2(1 + 2i) = (1^2 + 2 \cdot 1 \cdot 2i + (2i)^2)(1 + 2i)$   
 $= (1 + 4i + 4i^2)(1 + 2i) = (-3 + 4i)(1 + 2i) = -3 + 4i - 6i + 8i^2 = -11 - 2i$

2. (6pts)  $\frac{7+i}{3-5i} = \frac{7+i}{3-5i} \cdot \frac{3+5i}{3+5i} = \frac{21+3i+3i+5i^2}{3^2-(5i)^2} = \frac{2+38i-5}{9-25i^2} = \frac{16+38i}{34} = \frac{2(8+19i)}{34}$   
 $= \frac{8+19i}{17} = \frac{8}{17} + \frac{19}{17}i$

3. (4pts) Simplify and justify your answer.  
 $i^{163} = i^{160+3} = i^{160} \cdot i^3 = 1 \cdot i^3 = i^2 \cdot i = -i$   
 $(i^a)^{aa} = 1^{40} = 1$

4. (8pts) The number of smartphones in a warehouse (in thousands) is described by the function  $T(x) = x^2 - 14x + 70$ , where  $x$  is the number of days after March 25th.

- a) On what dates were there 37 thousand phones in the warehouse?  
 b) On what date did the number of smartphones in the warehouse reach its minimum?

a)  $x^2 - 14x + 70 = 37$      $x = 3$  Mar 28th    b)  $-\frac{b}{2a} = -\frac{-14}{2 \cdot 1} = 7$   
 $x^2 - 14x + 33 = 0$      $x = 11$  Apr 5th  
 $(x-3)(x-11) = 0$   
 $x = 3, 11$   
 April 1st

5. (8pts) Solve the equation:  $x^4 + 2x^2 - 63 = 0$   
 Let  $u = x^2$   
 $(x^2)^2 + 2x^2 - 63 = 0$   
 $u^2 + 2u - 63 = 0$   
 $(u+9)(u-7) = 0$   
 $u = -9, 7$   
 $x^2 = -9$      $x^2 = 7$   
 $x = \pm 3i$      $x = \pm \sqrt{7}$

6. (6pts) Solve by completing the square.

$x^2 + 10x + 30 = 0$      $+5^2$      $x + 5 = \pm \sqrt{5}i$   
 $x^2 + 2 \cdot x \cdot 5 + 5^2 + 30 = 5^2$      $-30$      $x = -5 \pm \sqrt{5}i$   
 $(x+5)^2 = -5$

7. (12pts) The quadratic function  $f(x) = -4x^2 + 4x + 15$  is given. Do the following without using the calculator.

- Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- Find the vertex of the graph.
- Sketch the graph of the function.

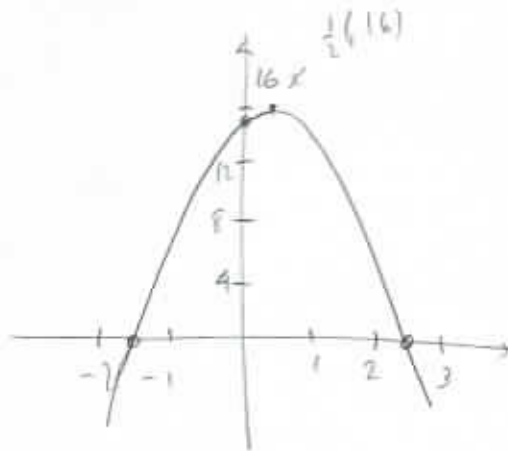
a)  $y$ -int:  $f(0) = 15$

$x$ -int:  $-4x^2 + 4x + 15 = 0$

$$4x^2 - 4x - 15 = 0 \quad 16 + 290$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot (-15)}}{2 \cdot 4}$$

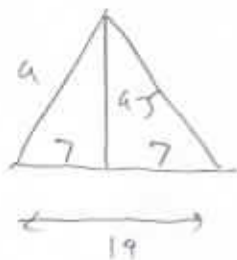
$$= \frac{4 \pm \sqrt{256}}{8} = \frac{4 \pm 16}{8} = \frac{20}{8}, \frac{-12}{8} = \frac{5}{2}, -\frac{3}{2}$$



b)  $h = -\frac{4}{2(-4)} = \frac{4}{8} = \frac{1}{2}$

$k = -4\left(\frac{1}{2}\right)^2 + 4 \cdot \frac{1}{2} + 15 = -4 \cdot \frac{1}{4} + 2 + 15 = 16$

8. (12pts) The base of an isosceles triangle is 14 cm (neither of the two equal sides is the base). If the height on that base is 5 cm shorter than the two equal sides of the triangle, what is the length of the equal sides?



$$(a-5)^2 + 7^2 = a^2$$

$$a^2 - 10a + 25 + 49 = a^2$$

$$-10a + 74 = 0$$

$$10a = 74$$

$$a = \frac{74}{10} = 7.4$$