

1. (21pts) For the following functions:  
 a) determine algebraically whether they are odd, even, or neither  
 b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^5 - 4x^3 + 5x$$

$$\begin{aligned} f(-x) &= (-x)^5 - 4(-x)^3 + 5(-x) \\ &= -x^5 - 4(-x^3) - 5x \\ &= -x^5 + 4x^3 - 5x = -f(x) \end{aligned}$$

odd



$$g(x) = x^4 - 3x^2 + 7$$

$$\begin{aligned} g(-x) &= (-x)^4 - 3(-x)^2 + 7 \\ &= x^4 - 3x^2 + 7 = g(x) \end{aligned}$$

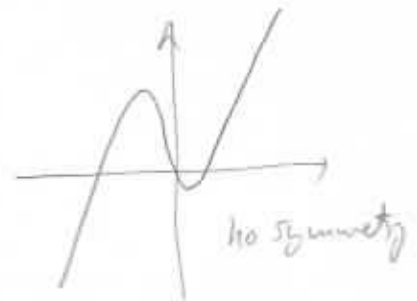
even



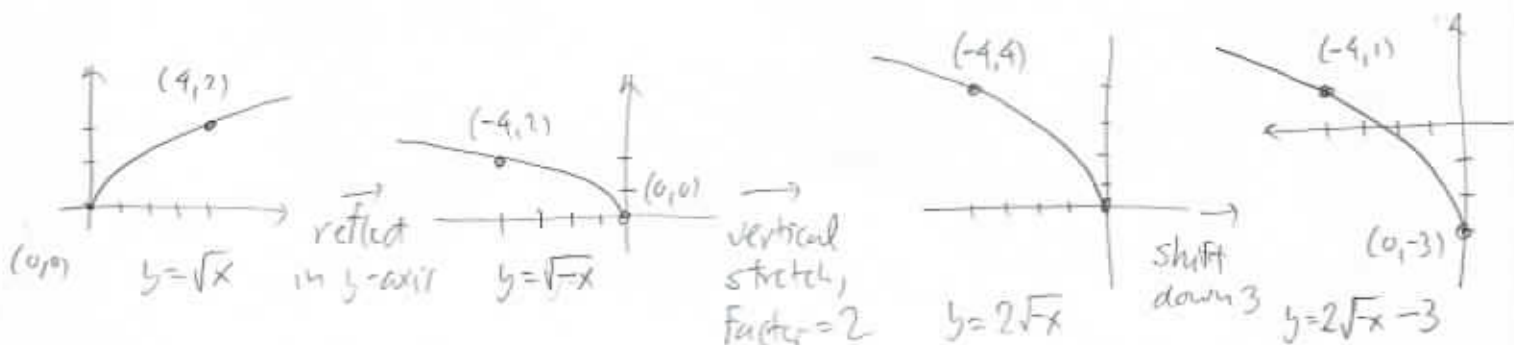
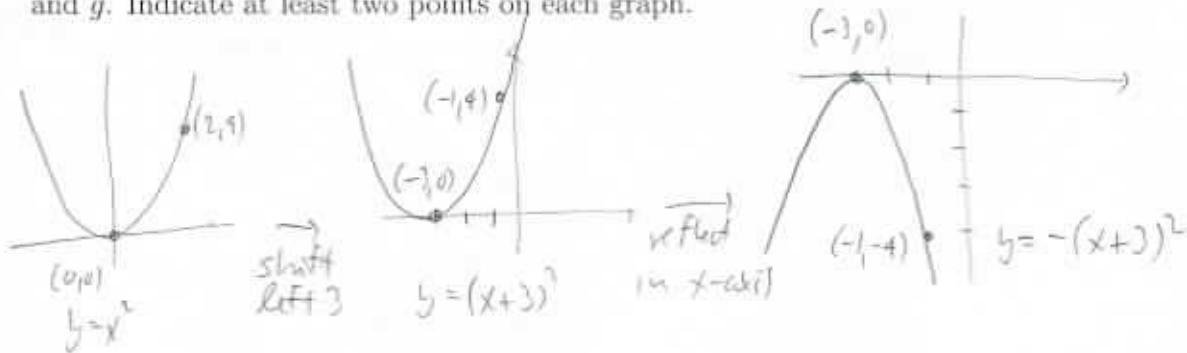
$$h(x) = x^3 + 3x^2 - 4x$$

$$\begin{aligned} h(-x) &= (-x)^3 + 3(-x)^2 - 4(-x) \\ &= -x^3 + 3x^2 + 4x \neq h(x) \\ &\neq -h(x) \end{aligned}$$

neither



2. (16pts) Draw the graphs of  $f(x) = -(x+3)^2$  and  $g(x) = -3 + 2\sqrt{-x}$  using transformations. Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.



3. (10pts) Write the equation for the function whose graph has the following characteristics:

a) shape of  $y = |x|$ , shifted right 4 units.

b) shape of  $y = x^3$ , stretched horizontally by factor 3, then reflected in  $x$ -axis

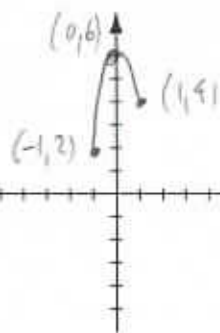
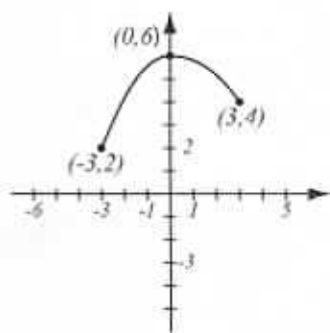
c) shape of  $y = \frac{1}{x}$ , reflected about the  $y$ -axis, then shifted left 3 units, then stretched vertically by factor 4.

a)  $y = |x| \rightsquigarrow y = |x - 4|$

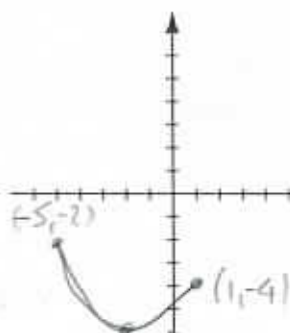
b)  $y = x^3 \rightsquigarrow y = \left(\frac{1}{3}x\right)^3 \rightsquigarrow y = -\left(\frac{1}{3}x\right)^3$

c)  $y = \frac{1}{x} \rightsquigarrow y = \frac{1}{-x} \rightsquigarrow y = \frac{1}{-(x+3)} \rightsquigarrow y = 4 \cdot \frac{1}{-(x+3)} = -\frac{4}{x+3}$

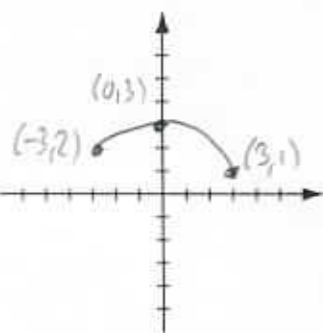
4. (13pts) The graph of  $f(x)$  is drawn below. On separate coordinate systems, sketch the graphs of the functions  $f(3x)$ ,  $-f(x+2)$  and  $\frac{1}{2}f(-x)$  and label all the relevant points.



horizontal stretch  
Factor =  $\frac{1}{3}$



shift left 2  
reflect in  $x$ -axis



reflect in  $y$ -axis  
vertical stretch factor =  $\frac{1}{2}$