College Algebra — Joysheet 5
MAT 140, Spring 2026 — D. Ivanšić

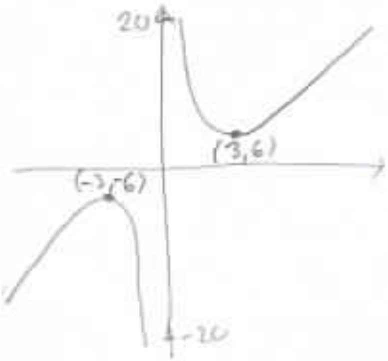
Name: Saul Ocean

Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{x^2+9}{x}$. (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) Local maximum is $-6 = f(-3)$
Local minimum is $6 = f(3)$

b) Increasing on $(-\infty,-3)$ and $(3,\infty)$
Decreasing on $(-3,0)$ and $(0,3)$

(Note function is not defined at $x=0$)

2. (20pts) Let $f(x) = \dfrac{x}{x^2-4}$, $g(x) = \sqrt{4x+13}$. Find the following (simplify where possible):

$(f+g)(3) = f(3)+g(3) = \dfrac{3}{3^2-4} + \sqrt{4\cdot 3+13}$
$= \dfrac{3}{5} + \sqrt{25} = \dfrac{3}{5}+5 = \dfrac{28}{5}$

$(fg)(7) = f(7)\cdot g(7) = \dfrac{7}{7^2-4}\cdot\sqrt{4\cdot 7+13}$
$= \dfrac{7}{45}\cdot\sqrt{41} = \dfrac{7\sqrt{41}}{45}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{x}{x^2-4}}{\sqrt{4x+13}} = \dfrac{x}{x^2-4}\cdot\dfrac{1}{\sqrt{4x+13}}$
$= \dfrac{x}{(x^2-4)\sqrt{4x+13}}$

$(g\circ f)(4) = g(f(4)) = g\left(\dfrac{4}{4^2-4}\right) = g\left(\dfrac{4}{12}\right)$
$= g\left(\dfrac{1}{3}\right) = \sqrt{4\cdot\frac{1}{3}+13} = \sqrt{\frac{4}{3}+13} = \sqrt{\frac{43}{3}}$

$(f\circ g)(x) = f(g(x)) = f(\sqrt{4x+13}) = \dfrac{\sqrt{4x+13}}{\sqrt{4x+13}^{\,2}-4} = \dfrac{\sqrt{4x+13}}{4x+13-4} = \dfrac{\sqrt{4x+13}}{4x+9}$

The domain of $(f-g)(x)$ in interval notation

Domain f: can't have $x^2-4=0$ $x^2=4$, $x=\pm 2$
Domain g: must have $4x+13\geq 0$ $x\geq -\frac{13}{4} = -3\frac{1}{4}$
$4x\geq -13$

-2 2 f
$-\frac{13}{4}$ g
$-13/4$ -2 2 f-g

$\left[-\frac{13}{4},-2\right)\cup(-2,2)\cup(2,\infty)$

3. (8pts) Consider the function $h(x) = \dfrac{5}{\sqrt{x-7}}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x-7$ $\qquad\qquad$ $g(x) = \sqrt{x-7}$

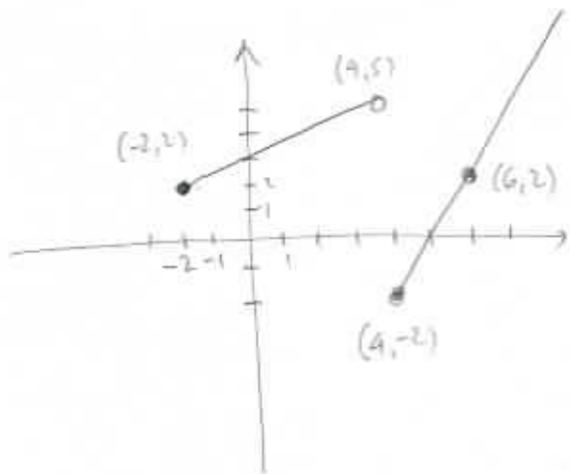
$f(x) = \dfrac{5}{\sqrt{x}}$ $\qquad\qquad$ $f(x) = \dfrac{5}{x}$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} \frac{1}{2}x + 3, & \text{if } -2 \le x < 4 \\ 2x - 10 & \text{if } x \ge 4 \end{cases}$$

x	$\frac{1}{2}x+3$
-2	2
4	5

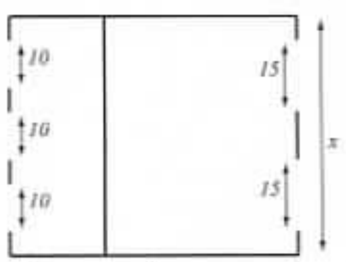
x	$2x-10$
4	-2
6	2



5. (14pts) Fulton University needs a garage for its maintenance vehicles with area 3000 square feet, divided into two sections, as in the picture. The sections have doors of width either 10 or 15 feet. The university wishes to minimize the construction cost, which is same as minimizing the total length of the walls.

a) Express the total length of the walls as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the garage for which the total length of the walls is minimal? What is the minimal wall length?



$L = x-30+x+x-30+y+y = 3x-60+2y = 3x-60+2\cdot\frac{3000}{x}$

$xy = 3000$ so $y = \frac{3000}{x}$

$L(x) = 3x + \frac{6000}{x} - 60$

$y = \frac{3000}{x} = \frac{3000}{44.721368}$

Dimensions: 44.721368, 67.082027

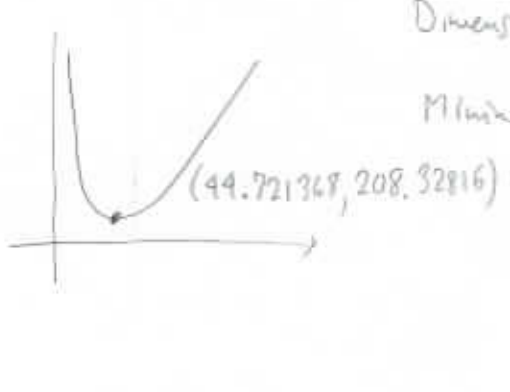
Minimal wall length: 208.32816

Domain: Must have

$x \geq 30$

$y > 0$

$\frac{3000}{x} > 0$ which is already true



Domain: $[30, \infty)$