

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_3 27 = 3 \quad \log_2 \frac{1}{16} = -4 \quad \log_a \sqrt[3]{a^5} = \frac{5}{3} \quad \log_{a^3} a^{12} = 4$$

$$3^? = 27 \quad 2^? = \frac{1}{16} = \frac{1}{2^4} = 2^{-4} \quad a^? = \sqrt[3]{a^5} = a^{\frac{5}{3}} \quad (a^3)^? = a^{12} = a^{3 \cdot 4} = (a^3)^4$$

2. (4pts) Use the change-of-base formula and your calculator to find  $\log_{12} 24$  with accuracy 6 decimal places. Show how you obtained your number.

$$\log_{12} 24 = \frac{\ln 24}{\ln 12} = 1.278943$$

3. (5pts) If  $\log_a 3 = 0.5283$  and  $\log_a 11 = 1.1531$ , calculate the following values:

$$\log_a \frac{3}{11} = \log_a 3 - \log_a 11$$

$$= 0.5283 - 1.1531$$

$$= -0.6248$$

$$\log_a 99 = \log_a (9 \cdot 11) = \log_a 9 + \log_a 11$$

$$= \log_a 3^2 + \log_a 11 = 2 \log_a 3 + \log_a 11$$

$$= 2 \cdot 0.5283 + 1.1531 = 2.2097$$

4. (4pts) Simplify.

$$\ln e^{\sqrt{8}} = \sqrt{8}$$

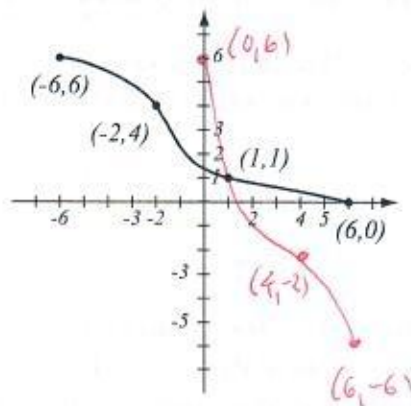
$$7^{\log_7(2u+5v)} = 2u+5v$$

5. (8pts) If you deposit \$8,000 in an account bearing 3.24% interest, compounded monthly, how much is in the account after 4 years?

$$A = P \left( 1 + \frac{r}{n} \right)^{nt} = 8000 \left( 1 + \frac{0.0324}{12} \right)^{12 \cdot 4} = 8000 \cdot 1.138... = 9105.39$$

6. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points, and showing any asymptotes.

b)



a) Yes - it passes the horizontal line test

7. (9pts) Let  $f(x) = \frac{2x-3}{x-5}$ .

- Find the formula for  $f^{-1}$ .
- Find the range of  $f$ .

$$y = \frac{2x-3}{x-5}$$

$$x = \frac{5y-3}{y-2}$$

$$y(x-5) = 2x-3$$

$$f^{-1}(y) = \frac{5y-3}{y-2}$$

$$yx - 5y = 2x - 3$$

$$yx - 2x - 5y = -3$$

$$yx - 2x = 5y - 3$$

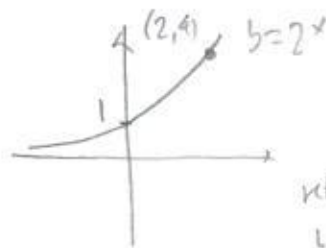
$$x(y-2) = 5y-3$$

Range  $f = \text{domain } f^{-1}$

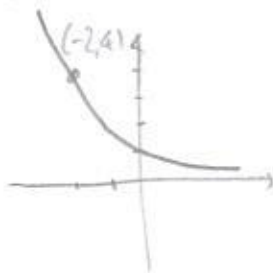
Can't have  $y-2=0$   
 $y=2$

$$\text{Range} = (-\infty, 2) \cup (2, \infty)$$

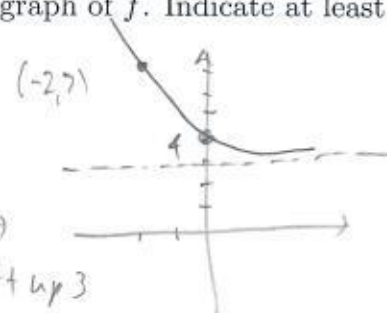
8. (6pts) Using transformations, draw the graph of  $f(x) = 3 + 2^{-x}$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Indicate at least one point on the graph and any asymptotes.



→ reflect in y-axis



→ shift up 3



9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_4 (16x^2 \sqrt[4]{y^7}) = \log_4 16 + \log_4 x^2 + \log_4 y^{\frac{7}{4}} = 2 + 2\log_4 x + \frac{7}{4}\log_4 y$$

$$\begin{aligned} \log_5 \frac{125y^6}{x^4y^2} &= \log_5 125 + \log_5 y^6 - \log_5 x^4 - \log_5 y^2 \\ &= 3 + 6\log_5 y - 4\log_5 x - 2\log_5 y \\ &= 3 + 4\log_5 y - 4\log_5 x \end{aligned}$$

10. (12pts) Write as a single logarithm. Simplify if possible.

$$\begin{aligned} 3\log(u^{-2}v^4) + 5\log(uv^2) &= \log (u^{-2}v^4)^3 + \log (uv^2)^5 \\ &= \log ((u^{-6}v^{12})(u^5v^{10})) = \log (u^{-1}v^{22}) = \log v^{22} - \log u \end{aligned}$$

$$\begin{aligned} 3\log_2(x^2 - 7x + 12) - 2\log_2(x-4) - \log_2(x-3) &= \log_2((x-3)(x-4))^3 - \log_2(x-4)^2 - \log_2(x-3) \\ &= \log_2 \frac{(x-3)^3(x-4)^3}{(x-4)^2(x-3)} = \log_2((x-3)^2(x-4)) \end{aligned}$$

Solve the equations.

11. (6pts)  $5^{2x-1} = 25^{4-3x}$

$$5^{2x-1} = (5^2)^{4-3x}$$

$$5^{2x-1} = 5^{8-6x}$$

$$2x-1 = 8-6x$$

$$8x = 9$$

$$x = \frac{9}{8}$$

12. (8pts)  $4^{x+1} = 5^{2-x}$  /  $\ln$

$$\ln 4^{x+1} = \ln 5^{2-x}$$

$$(x+1)\ln 4 = (2-x)\ln 5$$

$$x\ln 4 + \ln 4 = 2\ln 5 - x\ln 5$$

$$x\ln 4 + x\ln 5 = 2\ln 5 - \ln 4$$

$$x(\ln 4 + \ln 5) = 2\ln 5 - \ln 4$$

$$x = \frac{2\ln 5 - \ln 4}{\ln 4 + \ln 5} = 0.611731$$

13. (12pts) According to census data, the population of Lexington, KY, was 296,000 in 2010 and 323,000 in 2020. Assume that it has grown according to the formula  $P(t) = P_0 e^{kt}$ .

a) Find  $k$  and write the function that describes the population at time  $t$  years since 2010. Graph it on paper.

b) Find the predicted population in the year 2030.

$$a) P(t) = 296 e^{kt}$$

$$323 = P(10) = 296 e^{k \cdot 10}$$

$$323 = 296 e^{10k}$$

$$\frac{323}{296} = e^{10k} \quad | \ln$$

$$\ln \frac{323}{296} = 10k$$

$$k = \frac{\ln \frac{323}{296}}{10} = 0.00872929$$



$$b) P(20) = 296 e^{0.00872929 \cdot 20} = 352.462838$$

↑  
2030 is 20 years after 2010

About 352,463 are predicted for Lexington in 2030.

**Bonus** (10pts) Solve the equation.

$$3 \cdot 3^{2x} - 28 \cdot 3^x + 9 = 0$$

$$3(3^x)^2 - 28 \cdot 3^x + 9 = 0 \quad u = 3^x$$

$$3u^2 - 28u + 9 = 0$$

$$\text{prod} = 27 \quad -1, -27$$

$$\text{sum} = -28$$

$$3u^2 - 27u - u + 9 = 0$$

$$3u(u-9) - (u-9) = 0$$

$$(3u-1)(u-9) = 0$$

$$3u-1=0 \quad u-9=0$$

$$3u=1 \quad u=9$$

$$u = \frac{1}{3}$$

$$3^x = \frac{1}{3} = 3^{-1} \quad 3^x = 9$$

$$x = -1$$

$$x = 2$$