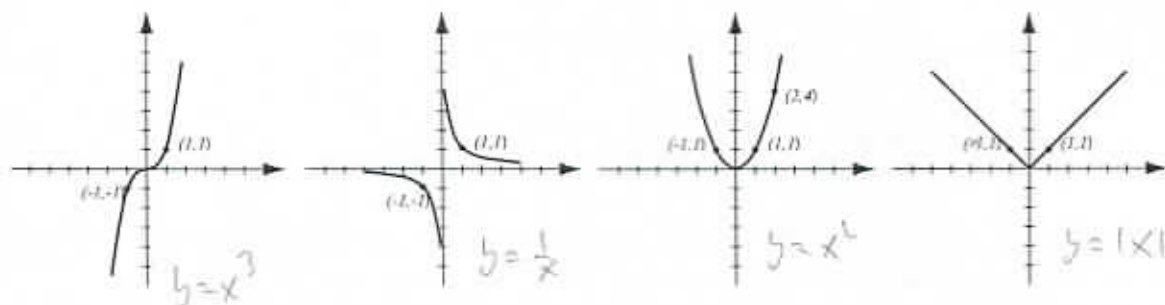


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let  $f(x) = \frac{1}{x^2 - 9}$ ,  $g(x) = x + 5$ .

Find the following (simplify where possible):

$$\begin{aligned} (f+g)(0) &= f(0) + g(0) = \frac{1}{0^2 - 9} + 0 + 5 \\ &= -\frac{1}{9} + 5 = \frac{44}{9} \end{aligned}$$

$$\begin{aligned} \frac{g}{f}(x) &= \frac{g(x)}{f(x)} = \frac{x+5}{\frac{1}{x^2-9}} = \frac{x+5}{1} \cdot \frac{x^2-9}{1} \\ &= (x+5)(x^2-9) = x^3 + 5x^2 - 9x - 45 \end{aligned}$$

$$\begin{aligned} (fg)(5) &= f(5) \cdot g(5) = \frac{1}{5^2 - 9} \cdot (5+5) \\ &= \frac{1}{16} \cdot 10 = \frac{10}{16} = \frac{5}{8} \end{aligned}$$

$$\begin{aligned} (g \circ f)(4) &= g\left(f(4)\right) = g\left(\frac{1}{4^2 - 9}\right) = g\left(\frac{1}{7}\right) \\ &= \frac{1}{7} + 5 = \frac{36}{7} \end{aligned}$$

$$(f \circ g)(x) = f(g(x)) = f(x+5) = \frac{1}{(x+5)^2 - 9} = \frac{1}{x^2 + 10x + 25 - 9} = \frac{1}{x^2 + 10x + 16}$$

The domain of  $fg$  in interval notation

domain  $f$ : can't have  $x^2 - 9 = 0$   
 $x^2 = 9$   
 $x = \pm 3$

domain  $g$ : all real numbers

~~Horizontal~~  $f$   
 $-3 \quad 3$   
~~Horizontal~~  $g$

~~Horizontal~~ domain  $fg$   
 $-3 \quad 3$   
 $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

3. (6pts) Consider the function  $h(x) = 3 + \sqrt{x+5}$  and find two different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

$$g(x) = \sqrt{x+5}$$

$$f(x) = x+5$$

$$f(x) = 3+x$$

$$g(x) = 3 + \sqrt{x}$$

4. (6pts) Write the equation for the function whose graph has the following characteristics:

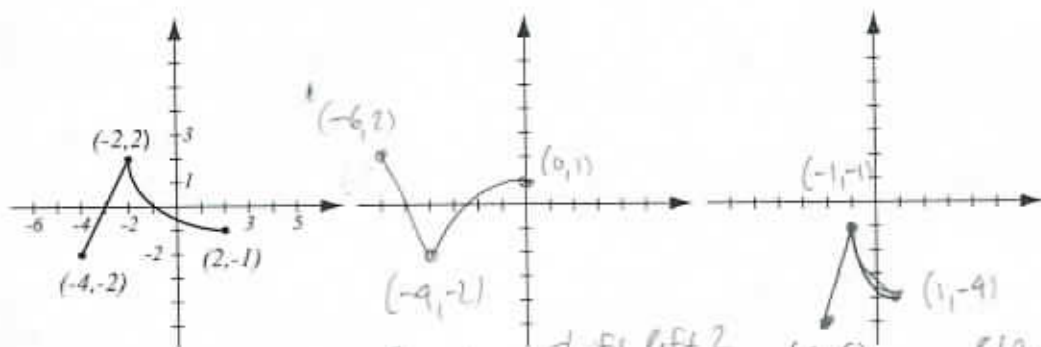
a) shape of  $y = x^2$ , stretched vertically by factor 3.

b) shape of  $y = x^3$ , shifted left by 4, then reflected over the  $y$ -axis.

$$a) y = x^2 \rightarrow y = 3x^2$$

$$b) y = x^3 \rightarrow y = (x+4)^3 \rightarrow y = (-x+4)^3$$

5. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $-f(x+2)$  and  $f(2x) - 3$  and label all the relevant points.



$$-f(x+2)$$

shift left 2  
reflect in  $x$ -axis

$$f(2x) - 3$$

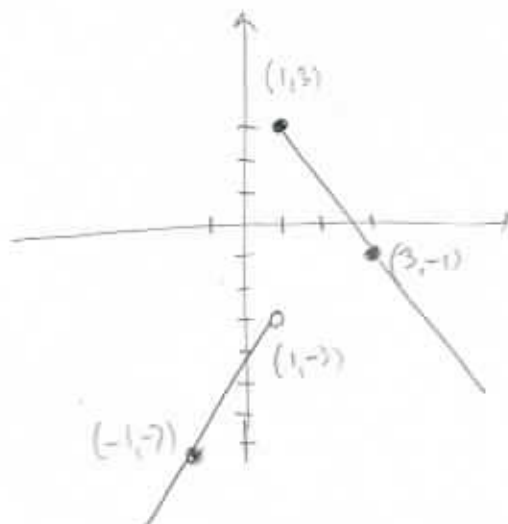
horiz. stretch, factor  $\frac{1}{2}$   
shift down 3

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x - 5, & \text{if } x < 1 \\ 5 - 2x, & \text{if } x \geq 1 \end{cases}$$

$x$	$2x-5$
1	-3
-1	-7

$x$	$5-2x$
1	3
3	-1



7. (5pts) Find the values of the piecewise-defined function.

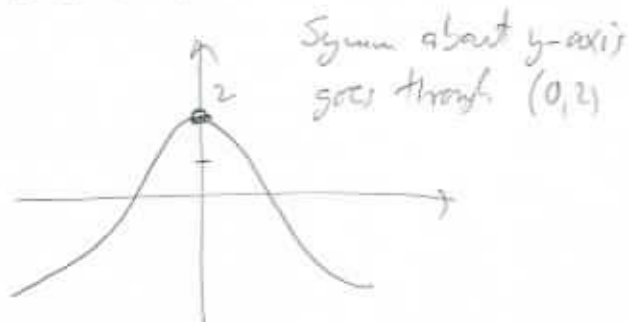
$$f(x) = \begin{cases} x^2 - 3x, & \text{if } -1 \leq x < 3 \\ \frac{4}{x^2 + 1}, & \text{if } x \geq 3 \end{cases}$$

$$f(7) = \frac{4}{7^2 + 1} = \frac{4}{50} = \frac{2}{25}$$

$$f(3) = \frac{4}{3^2 + 1} = \frac{4}{10} = \frac{2}{5}$$

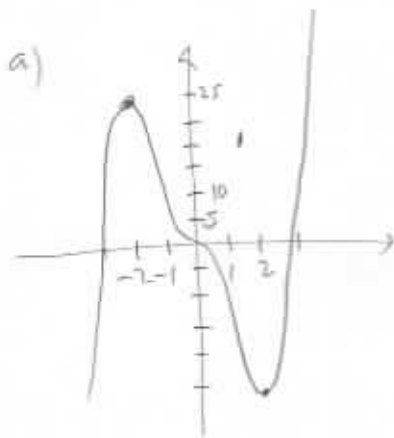
$$f(0) = 0^2 - 3 \cdot 0 = 0$$

8. (3pts) Sketch a graph of an even function with the property  $f(0) = 2$ . You can draw any curve you like, as long as it has the property requested.



9. (20pts) Let  $f(x) = x^5 - 7x^3$  (answer with 6 decimal points accuracy).

- Use your graphing calculator to accurately draw the graph of  $f$  (on paper!). Indicate units on the axes.
- Determine algebraically whether the function is odd, even, or neither.
- Verify your conclusion from b) by stating symmetry.
- Find the local maxima and minima for this function.
- State the intervals where the function is increasing and where it is decreasing.



$$\begin{aligned} b) \quad f(-x) &= (-x)^5 - 7(-x)^3 \\ &= -x^5 - 7(-x^3) \\ &= -x^5 + 7x^3 = -f(x) \end{aligned} \quad \text{odd function}$$

c) symm wrt origin

$$d) \text{ local max: } 24.100828 = f(-2.049393)$$

$$\text{local min: } -24.100828 = f(2.049393)$$

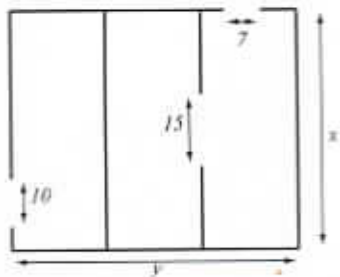
e) increasing on  $(-\infty, -2.049393)$  and  $(2.049393, \infty)$

decreasing on  $(-2.049393, 2.049393)$

10. (14pts) Landscape architect Zuri is designing a walled garden with area 4,000 square feet. Its plan is shown below, where the lines indicate walls. There are also three gaps in the walls to allow for entry into sections of the garden. Zuri's goal is to minimize construction cost, same as minimizing the total length of the walls.

a) Express the total wall length as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the enclosure that has the smallest total wall length and what is the smallest total wall length?



$$L = x - 10 + x + x - 15 + x + y + y - 7 = 4x + 2y - 32$$

$$xy = 4000 \text{ so } y = \frac{4000}{x}$$

$$L(x) = 4x + 2 \cdot \frac{4000}{x} - 32 = 4x + \frac{8000}{x} - 32$$

Domain:

must have:

$$x \geq 15$$

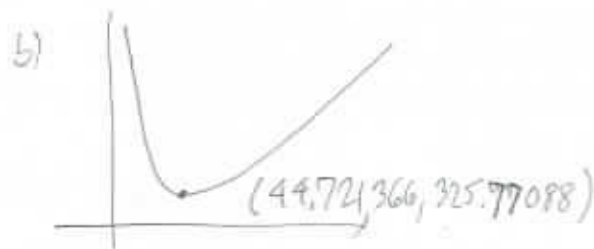
$$y \geq 7$$

$$\frac{4000}{x} \geq 7$$

$$4000 \geq 7x$$

$$x \leq \frac{4000}{7}$$

$$\left[15, \frac{4000}{7}\right]$$



$$\frac{4000}{44.721366}$$

Dimension: 44.721366 by 89.442707

Min. wall length 325.77088 ft

**Bonus.** (10pts) Among all right triangles whose hypotenuse has length 5, find the one that has the greatest area. You can use the two sides as base and height, because they are perpendicular. (Solve it like you did the optimization problem above.)



$$A = \frac{1}{2}xy$$

$$x^2 + y^2 = 25$$

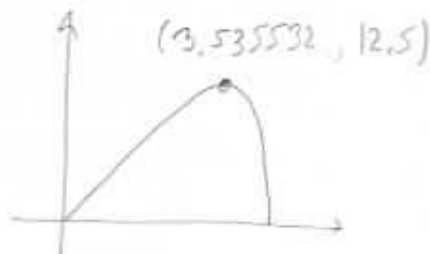
$$y^2 = 25 - x^2$$

$$y = \pm \sqrt{25 - x^2}$$

$$y = \sqrt{25 - x^2}$$

since  $y \geq 0$

$$A(x) = \frac{1}{2}x\sqrt{25 - x^2}$$



$$\sqrt{25 - 3.52^2}$$

dimension:  $x = 3.535532$  by  $y = 3.535532$

Max Area = 12.5