Integration Theory — Problem Farm 4.5
MAT 726, Spring 2025 — D. Ivanšić

Lebesgue Integrals

Type A problems (5pts each)

For the following functions and domains, determine if f is integrable over E. If it is, find the integral. Justify your reasoning with theorems, but if you do more than one of **A1–A5**, you may not appeal to the same theorem more than once.

A1. $f(x) = \dfrac{1}{x^p}$, $p > 1$, $E = [1, \infty)$

A2. $f(x) = \dfrac{1}{x^p}$, $0 < p \leq 1$, $E = [1, \infty)$

A3. $f(x) = \dfrac{1}{x^p}$, $p > 1$, $E = (0, 1]$

A4. $f(x) = \dfrac{1}{x^p}$, $0 < p \leq 1$, $E = (0, 1]$

A5. $f(x) = e^x$, $E = (-\infty, 0]$

A6. Show that $f(x) = \dfrac{1}{x^2} \sin x$ is integrable over $[1, \infty)$.

Type B problems (8pts each)

B1. Show that $f(x) = \dfrac{1}{x} \sin(\pi x)$ is not integrable over $[1, \infty)$, but that $\displaystyle\lim_{n \to \infty} \int_{[1,n]} f$ exists.