Trigonometry — Joysheet 1
MAT 145, Spring 2025 — D. Ivanšić

Name: _____

Covers: 6.1, 6.2 *Show all your work!*

1. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\tan\theta = \frac{7}{3}$.

2. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\sec\theta = w$, where w is some number.

3. (10pts) Given that $\sin 22^\circ = a$, $\tan 15^\circ = b$, $\csc 4^\circ = c$ and $\cot 70^\circ = d$, use basic and cofunction identities to express the following quantities using a, b, c and d.

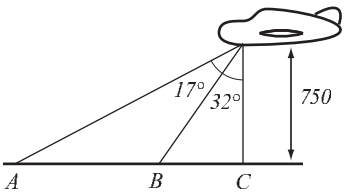
$\cot 75^\circ =$ $\sin 4^\circ =$ $\cos 68^\circ =$ $\tan 70^\circ =$

$\sec 86^\circ =$ $\sec 68^\circ =$ $\cot 20^\circ =$ $\cot 15^\circ =$

4. (10pts) Solve the right triangle (that is, find all sides and angles), if $a = 7$, $\alpha = 3°$.

5. (10pts) If the angle of elevation to the sun is $37°$, how long is the shadow of a 5.5 ft tall woman?

6. (14pts) An airplane flying over a straight road passes directly over point C at altitude 750 meters and observes that the angles subtending segments AB and BC are $17°$ and $32°$. How long are the segments AB and AC?



Trigonometry — Joysheet 2
MAT 145, Spring 2025 — D. Ivanšić

Name: _____

Covers: 6.3, 6.4 *Show all your work!*

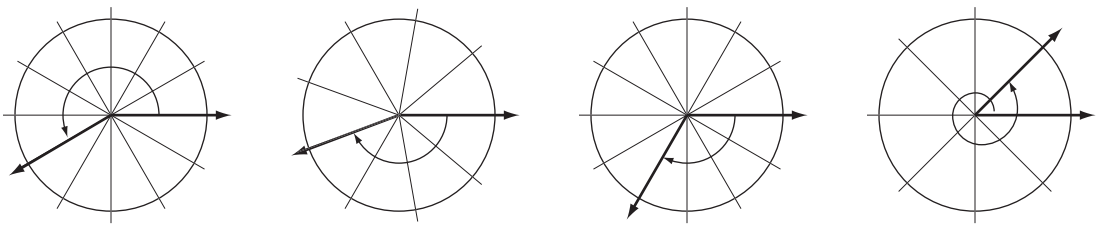
1. (9pts) If $\sin\theta = \dfrac{2}{3}$ and θ is in the second quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

2. (7pts) The terminal side of angle θ is in the third quadrant and lies on the line $3x-5y=0$. Find the exact values of $\cos\theta$ and $\cot\theta$. Draw a picture.

3. (8pts) Sketch angles in standard position with indicated radian measure.

$$\frac{4\pi}{3} \qquad -\frac{3\pi}{5} \qquad -\frac{5\pi}{4} \qquad \frac{17\pi}{7}$$

4. (8pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced lines to help you determine what the angles are.)



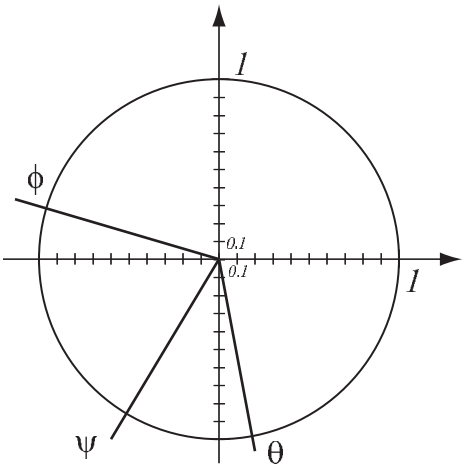
5. (8pts) What is the distance along Earth's surface from Murray, KY, latitude $36°36'34''$N, to the South Pole, if the radius of Earth is 3960 miles?

6. (8pts) A satellite is in geostationary orbit if it is always above a fixed point above the equator, which means it rotates around Earth at the same angular speed as Earth. This orbit is 26,200 miles from the Earth's center. What is the linear speed of a satellite in geostationary orbit?

7. (12pts) On a carousel, a rider sitting 6.75 meters from the center experiences linear speed of 8 kilometers per hour.
a) What is the angular speed of the carousel in radians per minute?
b) How many revolutions per minute does the carousel make?

Trigonometry — Joysheet 3 **MAT 145, Spring 2025 — D. Ivanšić**	**Name:** _____ *Covers: 6.5, 6.6* *Show all your work!*

1. (12pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn.



$\cos\psi =$ $\cot\psi =$

$\sin\phi =$ $\tan\phi =$

$\cos\theta =$ $\csc\theta =$

2. (18pts) For each of the following, draw the unit circle and the appropriate angle in order to infer from the picture the exact values of the trigonometric functions.

$\cos 225° =$ $\sin\dfrac{7\pi}{6} =$ $\sec(-90°) =$

$\tan(-210°) =$ $\csc\dfrac{5\pi}{3} =$ $\cot(5\pi) =$

$\sin(-420°) =$ $\tan\left(-\dfrac{11\pi}{4}\right) =$ $\cos\dfrac{35\pi}{6} =$

3. (8pts) Draw two periods of the graph of $y = 3\cos(2x)$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

4. (11pts) Draw two periods of the graph of $y = -2\sin\left(3x + \frac{\pi}{4}\right)$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

5. (11pts) Draw two periods of the graph of $y = 2\sin\left(-4x + \frac{\pi}{2}\right) - 1$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

Trigonometry — Joysheet 4
MAT 145, Spring 2025 — D. Ivanšić

Name: _____

Covers: 7.1, 7.2, 7.3 Show all your work!

1. (10pts) Suppose that $\frac{\pi}{2} < \alpha < \pi$ and $\frac{3\pi}{2} < \beta < 2\pi$ are angles so that $\sin\alpha = \frac{1}{4}$ and $\cos\beta = \frac{2}{7}$. Find the exact value of $\sin(\alpha - \beta)$.

2. (6pts) Show the identity in two ways:
1) algebraically 2) with a picture in which $\frac{\pi}{2} < \theta < \pi$

$\cos(\pi - \theta) = -\cos\theta$

3. (8pts) Use a half-angle formula to find the exact value of $\cos 105^\circ$ (do not use the calculator).

4. (8pts) Use identities to simplify the following expressions.

$$\cos\left(\frac{\pi}{2} - \theta\right)\sin\theta + \sin\left(\frac{\pi}{2} - \theta\right)\cos\theta =$$

$$\frac{\cot\left(\frac{\pi}{2} - \theta\right) + \tan\theta}{1 + \tan(-\theta)\cot\left(\frac{\pi}{2} - \theta\right)} =$$

5. (8pts) Show the identity.

$\sin\theta\cos(4\theta)\cos(2\theta) + \sin\theta\sin(4\theta)\sin(2\theta) - \cos\theta\sin(2\theta) = -\sin\theta$

6. (10pts) Show the identity.

$$\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\sin(2\theta)}{\cos(2\theta)}$$

7. (10pts) Develop the formula for $\cos(4\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$\cos(4\theta) = \cos(2\cdot(2\theta)) =$

Trigonometry — Joysheet 5
MAT 145, Spring 2025 — D. Ivanšić

Name: _____

Covers: 7.4, 7.5 *Show all your work!*

1. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arcsin \dfrac{\sqrt{3}}{2} =$ $\arccos\left(-\dfrac{\sqrt{3}}{2}\right) =$ $\arctan\left(-\dfrac{1}{\sqrt{3}}\right) =$ $\arcsin(3) =$

2. (7pts) Find the exact value of the expressions (do not use the calculator). For some of them, you will need a picture.

$\tan(\arctan 7.3) =$ $\arcsin\left(\sin\left(-\dfrac{2\pi}{5}\right)\right) =$ $\arccos\left(\cos\dfrac{9\pi}{7}\right) =$

3. (5pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$\sec\left(\arcsin\dfrac{2}{3}\right) =$

4. (5pts) Solve the equation (give a general formula for all solutions).

$2\cos\theta + 1 = 0$

5. (5pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.3$

6. (10pts) Solve the equation and give a general formula for all solutions. Then list all the solutions that fall in the interval $[0, 2\pi)$.

$2\sin^2\theta - 7\sin\theta - 4 = 0$

7. (6pts) Solve the equation on the interval $[0, 2\pi)$.

$\cos(2\theta) + 2\cos^2\theta = 1$

8. (7pts) Solve the equation (give a general formula for all the solutions).

$\sec^2\theta + \tan^2\theta = \tan\theta + 2$

9. (7pts) Find the exact value of the expression (do not use the calculator).

$$\sin\left(\frac{\pi}{3} + \arcsin\left(-\frac{1}{4}\right)\right) =$$

Trigonometry — Joysheet 6
MAT 145, Spring 2025 — D. Ivanšić

Name: _____

Covers: 8.1, 8.2 *Show all your work!*

1. (7pts) Solve the triangle: $a = 5$, $C = 28°$, $A = 50°$.

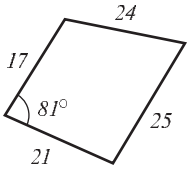
2. (12pts) Solve the triangle: $c = 9$, $a = 8$, $A = 53°$

3. (7pts) Solve the triangle: $a = 5$, $b = 3$, $c = 6$.

4. (10pts) A straight bridge over a canyon is 247 meters long. Observed from the ends of the bridge, the bottom of the canyon has angles of depression 76° and 68°. How high above the bottom of the canyon is the bridge? (Draw a picture.)

5. (14pts) Airport B is located 89 miles N36°W from airport A. A plane is 121 miles from airport B and has position N25°E of airport A. How far is the plane from airport A (draw a picture)?

6. (10pts) Determine the area of the pictured quadrangle.



Trigonometry — Joysheet 7
MAT 145, Spring 2025 — D. Ivanšić

Name: _____

Covers: 8.4 *Show all your work!*

1. (12pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(2, \frac{4\pi}{3}\right)$ $(r, \theta) = \left(-1, \frac{7\pi}{4}\right)$ $(r, \theta) = \left(-3, -\frac{7\pi}{2}\right)$

2. (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$(x, y) = (-2, 2)$ $(x, y) = (-3, -\sqrt{3})$ $(x, y) = (-2, 3)$

3. (6pts) Convert to a polar equation.

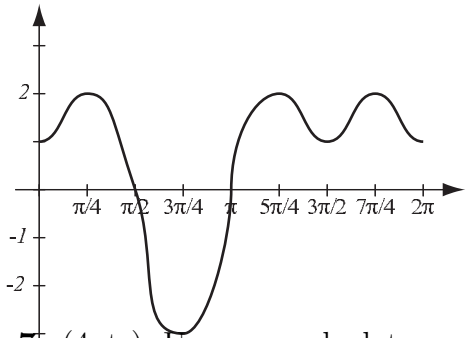
$x^4 - x^2y^2 + y^4 = 4$

4. (6pts) Convert to a rectangular equation.

$r = 1 + \cos\theta$

5. (12pts) Graph the equation $r = 1 - 2\cos(\theta)$ by doing the following:
a) Graph the equation in rectangular r-θ coordinates.
b) Use the information from a) to help you graph the equation in polar coordinates. Indicate corresponding parts of the graph in a) and b). Check your work with the graphing calculator.

6. (8pts) Below is the graph of the function $r = f(\theta)$ in rectangular $r - \theta$ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates indicating corresponding parts of the graphs.



7. (4pts) Use your calculator or a graphing program (look online) to draw accurate graphs of the polar curve.

$r = 2 - 3\sin(4\theta)$