Trigonometry — Joysheet 7
MAT 145, Spring 2025 — D. Ivanšić

Name: Saul Ocean

Covers: 8.4 *Show all your work!*

1. (12pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r,\theta) = \left(2, \frac{4\pi}{3}\right)$

$x = 2\cos\frac{4\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

$y = 2\sin\frac{4\pi}{3} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$(r,\theta) = \left(-1, \frac{7\pi}{4}\right)$

$x = -1\cos\frac{7\pi}{4} = -1\cdot\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

$y = -1\sin\frac{7\pi}{4} = -1\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$

$(r,\theta) = \left(-3, -\frac{7\pi}{2}\right)$

$x = -3\cos\left(-\frac{7\pi}{2}\right) = -3\cdot 0 = 0$

$y = -3\sin\left(-\frac{7\pi}{2}\right) = -3\cdot -1 = -3$

2. (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$(x,y) = (-2,2)$

$r = \sqrt{(-2)^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{2}{-2} = -1$

$\theta = \frac{3\pi}{4}$

$\left(2\sqrt{2}, \frac{3\pi}{4}\right)$

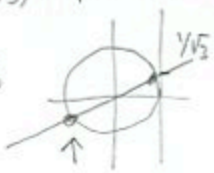
$\left(2\sqrt{2}, -\frac{5\pi}{4}\right)$

$\left(-2\sqrt{2}, -\frac{\pi}{4}\right)$

$(x,y) = (-3,-\sqrt{3})$

$r = \sqrt{(-3)^2+(-\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$

$\tan\theta = \frac{-\sqrt{3}}{-3} = \frac{1}{\sqrt{3}}$



$\theta = \frac{7\pi}{6}$

$\left(2\sqrt{3}, \frac{7\pi}{6}\right)$

$\left(2\sqrt{3}, -\frac{5\pi}{6}\right)$

$\left(-2\sqrt{3}, \frac{\pi}{6}\right)$

$(x,y) = (-2,3)$

$r = \sqrt{(-2)^2+3^2} = \sqrt{13}$

$\tan\theta = \frac{3}{-2} = -\frac{3}{2}$

$\theta = \arctan\left(-\frac{3}{2}\right) + \pi$

$\left(\sqrt{13}, \arctan\left(-\frac{3}{2}\right)+\pi\right)$

$\left(\sqrt{13}, \arctan\left(-\frac{3}{2}\right)-\pi\right)$

$\left(-\sqrt{13}, \arctan\left(-\frac{3}{2}\right)\right)$

3. (6pts) Convert to a polar equation.

$x^4 - x^2y^2 + y^4 = 4$

$(r\cos\theta)^4 - (r\cos\theta)^2(r\sin\theta)^2 + (r\sin\theta)^4 = 4$

$r^4\cos^4\theta - r^4\cos^2\theta\sin^2\theta + r^4\sin^4\theta = 4$

$r^4\left(\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta\right) = 4$

$r = \pm\sqrt[4]{\frac{4}{\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta}}$

also equal to $(\cos^2\theta+\sin^2\theta)^2 - 3\cos^2\theta\sin^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

4. (6pts) Convert to a rectangular equation.

$r = 1 + \cos\theta \quad | \cdot r$

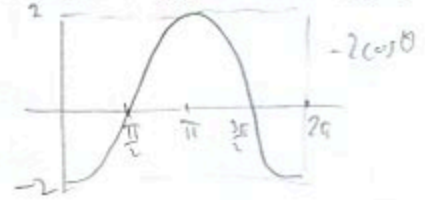
$r^2 = r + r\cos\theta$

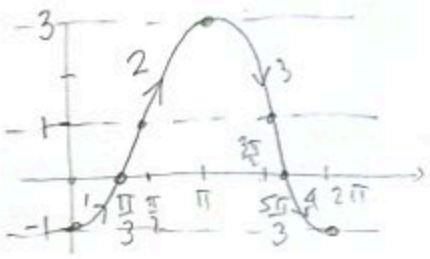
$x^2+y^2 = \sqrt{x^2+y^2} + x$

$x^2+y^2-x = \sqrt{x^2+y^2}$

$\left(x^2+y^2-x\right)^2 = x^2+y^2$

5. (12pts) Graph the equation $r = 1 - 2\cos(\theta)$ by doing the following:
a) Graph the equation in rectangular r-θ coordinates.
b) Use the information from a) to help you graph the equation in polar coordinates. Indicate corresponding parts of the graph in a) and b). Check your work with the graphing calculator.

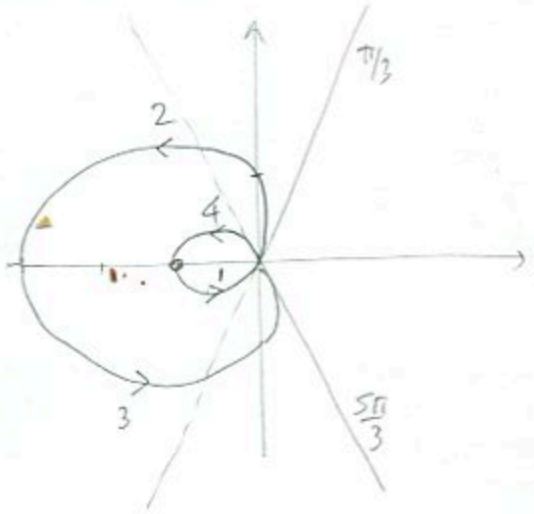




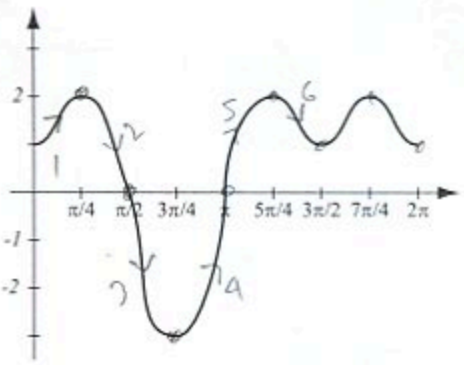
$1-2\cos\theta=0$

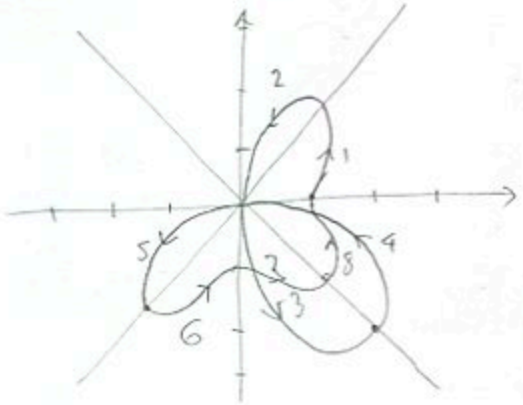
$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$



6. (8pts) Below is the graph of the function $r = f(\theta)$ in rectangular $r - \theta$ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates indicating corresponding parts of the graphs.





7. (4pts) Use your calculator or a graphing program (look online) to draw accurate graphs of the polar curve.

$r = 2 - 3\sin(4\theta)$