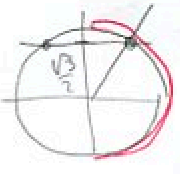
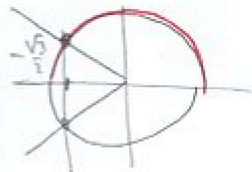
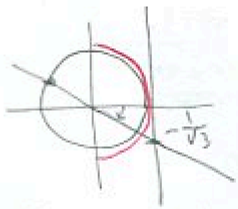
Trigonometry — Joysheet 5	**Name:** Saul Ocean
MAT 145, Spring 2025 — D. Ivanšić	*Covers: 7.4, 7.5* *Show all your work!*

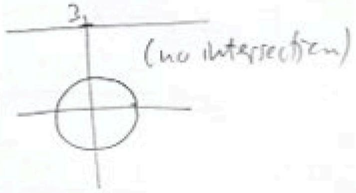
1. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ $\arcsin(3) =$ not defined





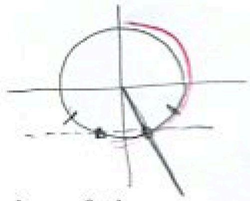


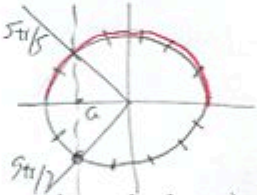


2. (7pts) Find the exact value of the expressions (do not use the calculator). For some of them, you will need a picture.

$\tan(\arctan 7.3) = 7.3$ $\arcsin\left(\sin\left(-\frac{2\pi}{5}\right)\right) = -\frac{2\pi}{5}$ $\arccos\left(\cos\frac{9\pi}{7}\right) = \arccos a$

$= \frac{5\pi}{7}$





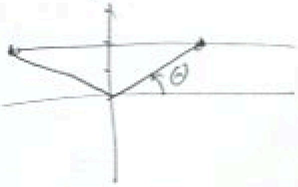
3. (5pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$\sec\left(\arcsin\frac{2}{3}\right) = \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}$

$\theta = \arcsin\frac{2}{3}$

$\sin\theta = \frac{2}{3}$

and θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$x^2 + 2^2 = 3^2$

$x^2 = 9 - 4$

$x = \pm\sqrt{5} = \sqrt{5}$

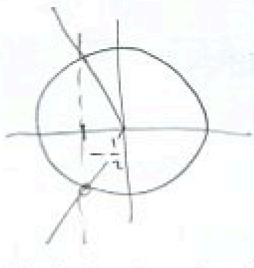
since $x \geq 0$

4. (5pts) Solve the equation (give a general formula for all solutions).

$2\cos\theta + 1 = 0$

$2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

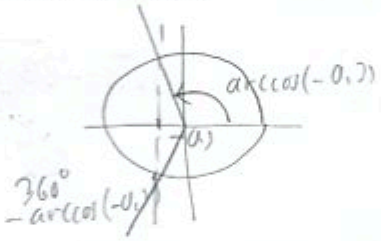


$\theta = \frac{2\pi}{3} + k\cdot 2\pi$

$-\frac{2\pi}{3} + k\cdot 2\pi$

5. (5pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = -0.3$



sol. are $\arccos(-0.3)$, $360° - \arccos(-0.3)$

$= 107.457603°$, 252.542397

6. (10pts) Solve the equation and give a general formula for all solutions. Then list all the solutions that fall in the interval $[0, 2\pi)$.

$2\sin^2\theta - 7\sin\theta - 4 = 0$

Let $u = \sin\theta$

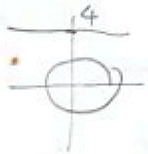
$2u^2 - 7u - 4 = 0$ $\quad 49+32$

$u = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4\cdot 2\cdot(-4)}}{2\cdot 2}$

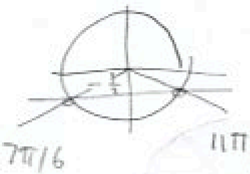
$= \dfrac{7 \pm \sqrt{81}}{4} = \dfrac{7\pm 9}{4} = \dfrac{16}{4}, -\dfrac{2}{4} = 4, -\dfrac{1}{2}$

$\sin\theta = 4$

no sol



$\sin\theta = -\frac{1}{2}$



$\theta = -\frac{\pi}{6} + k\cdot 2\pi$

$\theta = -\frac{5\pi}{6} + k\cdot 2\pi$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

7. (6pts) Solve the equation on the interval $[0, 2\pi)$.

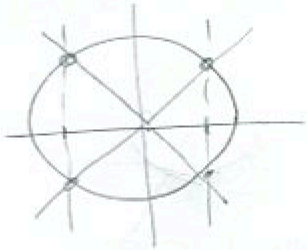
$\cos(2\theta) + 2\cos^2\theta = 1$

$2\cos^2\theta - 1 + 2\cos^2\theta = 1$

$4\cos^2\theta = 2$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$



$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

8. (7pts) Solve the equation (give a general formula for all the solutions).

$\sec^2\theta + \tan^2\theta = \tan\theta + 2$

$\tan^2\theta + 1 + \tan^2\theta = \tan\theta + 2$

$2\tan^2\theta - \tan\theta - 1 = 0$

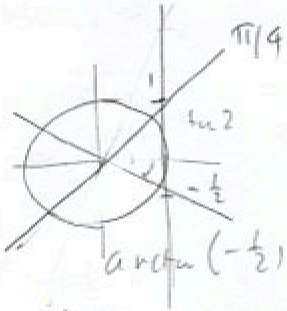
let $u = \tan\theta$

$2u^2 - u - 1 = 0$

$u = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4\cdot 2\cdot(-1)}}{2\cdot 2}$ $\quad 1+8$

$= \dfrac{1\pm\sqrt{9}}{4} = \dfrac{1\pm 3}{4} = 1, -\frac{1}{2}$

$\tan\theta = 1$ or $\tan\theta = -\frac{1}{2}$



$\theta = \frac{\pi}{4} + k\pi$ or $\theta = \arctan(-\frac{1}{2}) + k\cdot\pi$

9. (7pts) Find the exact value of the expression (do not use the calculator).

$\sin\left(\dfrac{\pi}{3} + \arcsin\left(-\dfrac{1}{4}\right)\right) = \sin\frac{\pi}{3}\cos\arcsin(-\frac{1}{4}) + \cos\frac{\pi}{3}\sin(\arcsin(-\frac{1}{4}))$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{15}}{4} + \frac{1}{2}\left(-\frac{1}{4}\right) = \dfrac{3\sqrt{5} - 1}{8}$

$\sqrt{3}\sqrt{15} = \sqrt{3\cdot 3\cdot 5} = 3\sqrt{5}$

$\theta = \arcsin(-\frac{1}{4})$

$\sin\theta = -\frac{1}{4}$ θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$x^2 + (-1)^2 = 4^2$

$x^2 = 16 - 1$

$x = \pm\sqrt{15}$ $\quad x = \sqrt{15}$

$\cos\theta = \frac{\sqrt{15}}{4}$