Trigonometry — Joysheet 4
MAT 145, Spring 2025 — D. Ivanšić

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Covers: 7.1, 7.2, 7.3 Show all your work!

1. (10pts) Suppose that $\frac{\pi}{2} < \alpha < \pi$ and $\frac{3\pi}{2} < \beta < 2\pi$ are angles so that $\sin\alpha = \frac{1}{4}$ and $\cos\beta = \frac{2}{7}$. Find the exact value of $\sin(\alpha - \beta)$.

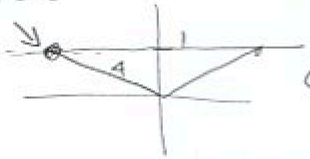
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{1}{4}\cdot\frac{2}{7} - \frac{-\sqrt{15}}{4}\cdot\frac{(-3\sqrt{5})}{7} = \frac{2-3\sqrt{3\cdot 25}}{28}$

$= \frac{2-15\sqrt{3}}{28}$

$\sin\alpha = \frac{1}{4} = \frac{y}{r}$

$\cos\beta = \frac{2}{7} = \frac{x}{r}$

this one



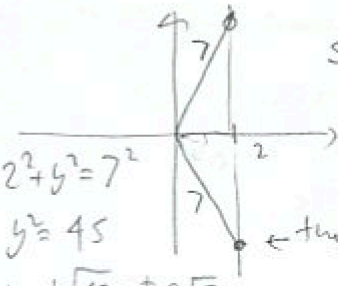
$\cos\alpha = \frac{-\sqrt{15}}{4}$

$x^2+1^2=4^2$

$x^2=15$

$x=\pm\sqrt{15}$

$x=-\sqrt{15}$ due to $\frac{\pi}{2}<\alpha<\pi$



← this one

$\sin\beta = \frac{-3\sqrt{5}}{7}$

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$2^2+y^2=7^2$

$y^2=45$

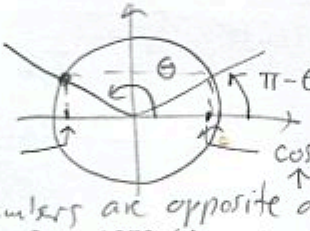
$y=\pm\sqrt{45}=\pm 3\sqrt{5}$

$y=-3\sqrt{5}$ due to $\frac{3\pi}{2}<\beta<2\pi$

2. (6pts) Show the identity in two ways:
1) algebraically 2) with a picture in which $\frac{\pi}{2} < \theta < \pi$

$\cos(\pi - \theta) = -\cos\theta$

$\underbrace{\cos\pi}_{-1}\cos\theta + \underbrace{\sin\pi}_{0}\sin\theta = -\cos\theta$

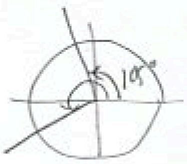


cos θ ↑ cos(π−θ) ↑

these numbers are opposite due to symmetry

3. (8pts) Use a half-angle formula to find the exact value of $\cos 105°$ (do not use the calculator).

$\cos^2 105° = \frac{1+\cos 210°}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2}\cdot\frac{2}{2} = \frac{2-\sqrt{3}}{4}$



210°

$\cos 105° = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

↑ since 105° is in 2nd quadrant

4. (8pts) Use identities to simplify the following expressions.

$\cos\left(\frac{\pi}{2}-\theta\right)\sin\theta + \sin\left(\frac{\pi}{2}-\theta\right)\cos\theta = \sin\theta\sin\theta + \cos\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta = 1$

$\frac{\cot\left(\frac{\pi}{2}-\theta\right)+\tan\theta}{1+\tan(-\theta)\cot\left(\frac{\pi}{2}-\theta\right)} = \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta} = \frac{2\tan\theta}{1-\tan^2\theta} = \tan(2\theta)$

5. (8pts) Show the identity.

$$\underbrace{\sin\theta\cos(4\theta)\cos(2\theta)+\sin\theta\sin(4\theta)\sin(2\theta)-\cos\theta\sin(2\theta)}=-\sin\theta$$

$$= \sin\theta\,\big(\underbrace{\cos(4\theta)\cos(2\theta)+\sin(4\theta)\sin(2\theta)}\big) - \cos\theta\sin(2\theta)$$

$$= \sin\theta\ \cos(4\theta-2\theta) - \cos\theta\sin(2\theta)$$

$$= \sin\theta\cos(2\theta) - \cos\theta\sin(2\theta) = \sin(\theta-2\theta)$$

$$= \sin(-\theta) = -\sin\theta$$

6. (10pts) Show the identity.

$$\frac{1+\tan\theta}{1-\tan\theta}=\frac{1+\sin(2\theta)}{\cos(2\theta)}$$

$$\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}\cdot\frac{\cos\theta}{\cos\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\cdot\frac{\cos\theta+\sin\theta}{\cos\theta+\sin\theta}$$

$$= \frac{(\cos\theta+\sin\theta)^2}{\cos^2\theta-\sin^2\theta} = \frac{\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta}{\cos(2\theta)}$$

$$= \frac{1+2\sin\theta\cos\theta}{\cos(2\theta)} = \frac{1+\sin(2\theta)}{\cos(2\theta)}$$

7. (10pts) Develop the formula for $\cos(4\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$$\cos(4\theta)=\cos(2\cdot(2\theta)) = \cos^2(2\theta)-\sin^2(2\theta) = (\cos^2\theta-\sin^2\theta)^2-(2\sin\theta\cos\theta)^2$$

$$= \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta - 4\sin^2\theta\cos^2\theta$$

$$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$$

OR $= 2\cos^2(2\theta)-1 = 2(2\cos^2\theta-1)^2-1$

$$= 2(4\cos^4\theta-4\cos^2\theta+1)-1 = 8\cos^4\theta-8\cos^2\theta+1$$

OR $= 1-2\sin^2(2\theta) = 1-2(2\sin\theta\cos\theta)^2 = 1-8\sin^2\theta\cos^2\theta$