**Trigonometry — Exam 1**
**MAT 145, Spring 2025— D. Ivanšić**

**Name:** _____

*Show all your work!*

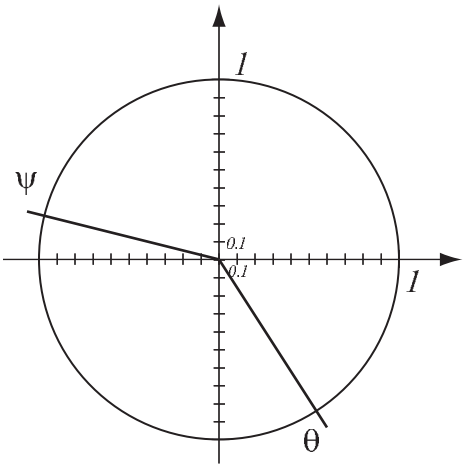
**1.** (10pts) If $\theta$ is an acute angle, find the values of all the trigonometric functions of $\theta$ given that $\cos\theta = \frac{2}{7}$. Draw a picture.

**2.** (12pts) If $\sin\theta = \frac{3}{4}$ and $\theta$ is in the second quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.

**3.** (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\sin(-135^\circ) =$ $\cos\dfrac{4\pi}{3} =$ $\sec(210^\circ) =$ $\tan\dfrac{9\pi}{2} =$

**4.** (9pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn. Note the angles are **not** the standard angles.




$\sin\theta =$ $\cos\theta =$

$\sec\psi =$ $\tan\psi =$

**5.** (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$25^\circ =$

$\frac{13\pi}{9}$ radians $=$

**6.** (9pts) Use reciprocal identities and complementary angle identities to simplify:

$\frac{\cos 32^\circ}{\cos 58^\circ} - \tan 58^\circ =$

$\cos 43^\circ \sec 43^\circ + \cos 29^\circ \csc 61^\circ =$

**7.** (10pts) Draw two periods of the graph of $y = 2\cos\left(3x + \frac{\pi}{4}\right)$.
What is the amplitude? The period?
For each period, indicate $x$-coordinates of the five special points (middle, peaks, valleys).

**8.** (10pts) A ski slope has angle $12^\circ$ with the horizontal. If a skier going straight down the hill travels 120 meters, what is the amount of her vertical descent?

**9.** (10pts) You wrap a 1-yard string around a wheel with radius 15 inches. Consider the angle whose tip is at the center of the wheel and whose sides go through the ends of the string on the wheel. What is the size of the angle in radians? In degrees?

**10.** (12pts) The St. Louis ferris wheel is 60 meters in diameter and takes 5 minutes to complete one revolution.
a) What is the wheel's angular velocity in radians per second?
b) What linear velocity does a passenger experience in meters per second?

**Bonus.** (10pts) Standing on one side of a building, Jayden measures the angle of elevation to the top of the building to be $50°$. He walks toward the building, goes through a corridor in the building, emerges on the other side and continues walking away from the building, all in a straight line. When Jayden stops, he turns around and measures the angle of elevation to the top of the building to be $38°$. If the measuring wheel he had with him says he walked a total of 87 meters, how tall is the building?

**Trigonometry — Exam 2**
**MAT 145, Spring 2025— D. Ivanšić**

**Name:** _____

*Show all your work!*

| | |
|---|---|
| $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ | $\sin(2u) = 2 \sin u \cos u$ |
| $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ | $\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ |
| $\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ | $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$ |
| $\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ | $\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$ |

**1.** (16pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$\sin \dfrac{11\pi}{12} =$

$\cos 67.5^\circ =$

**2.** (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arccos(-3) =$ $\qquad$ $\arccos \dfrac{\sqrt{3}}{2} =$ $\qquad$ $\arcsin\left(-\dfrac{1}{2}\right) =$ $\qquad$ $\arctan(-1) =$

**3.** (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\tan(\arctan 3.7) =$ $\qquad$ $\arcsin\left(\sin \dfrac{7\pi}{5}\right) =$

**4.** (6pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\tan\left(\arccos\left(-\frac{3}{4}\right)\right) =$$

**5.** (8pts) Use identities to simplify the following expression.

$$\left(\sin\left(\frac{\pi}{2}-\theta\right)+\sin\theta\right)\left(\cos\theta-\cos\left(\frac{\pi}{2}-\theta\right)\right) =$$

Show the identities:

**6.** (8pts) $\sin\theta(\csc\theta-\sin\theta)=\cos^2\theta$

**7.** (10pts) $\tan\theta+\cot\theta=\dfrac{2}{\sin(2\theta)}$

**8.** (5pts) Solve the equation in radians (state general solution).

$4\sin\theta - 2\sqrt{2} = 0$

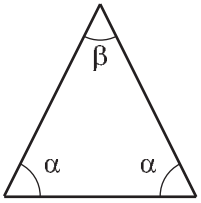
**9.** (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\sin\theta = -0.33$

**10.** (14pts) Solve the equation in radians.
a) State the general solution.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$6\cos^2\theta - \cos\theta - 1 = 0$

**11.** (12pts) An isosceles triangle has two sides of equal length and two angles of same measure. If an isosceles triangle has angles $\alpha$, $\alpha$ and $\beta$, and $\cos\alpha = \frac{1}{3}$, find the exact value of $\cos\beta$ (do not use the calculator).




**Bonus.** (10pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$ are angles so that $\cos\alpha = -\frac{2}{5}$ and $\cos\beta = \frac{2}{7}$. Find the exact value of $\sin(2\alpha + 2\beta)$.

**Trigonometry — Exam 3**
**MAT 145, Spring 2025— D. Ivanšić**

**Name:** _____

*Show all your work!*

| | |
|---|---|
| $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ | $\sin(2u) = 2 \sin u \cos u$ |
| $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ | $\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ |
| $\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ | $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$ |
| $\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ | $\sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ $\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$ |

**1.** (6pts) Solve the triangle: $b = 5$, $c = 3$, $C = 40°$.

**2.** (12pts) Solve the triangle: $a = 4$, $c = 7$, $B = 112°$

**3.** (14pts) Solve the triangle: $a = 7$, $c = 2$, $A = 67°$.

**4.** (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(4, \frac{3\pi}{4}\right)$ $(r, \theta) = \left(-2, -\frac{7\pi}{6}\right)$

**5.** (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct $\theta$. For each point, give three answers in polar coordinates, at least one of which has a negative $r$. Give exact answers — do not use the calculator.

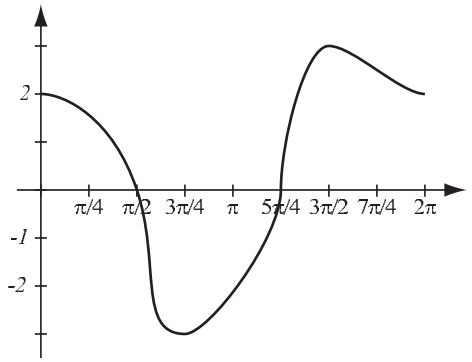
$(x, y) = (-5, -5)$ $(x, y) = (3\sqrt{3}, -3)$

**6.** (8pts) Convert to a polar equation. Answer should be solved for $r$.

$x^2 - 2x + y^2 = 1$

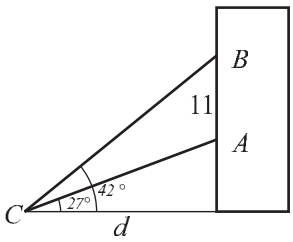
**7.** (8pts) Determine the distance between points given in **polar coordinates**: $B = \left(4, \frac{2\pi}{3}\right)$, $C = \left(2, \frac{\pi}{3}\right)$.
a) Draw the picture.
b) Find the exact distance from $B$ to $C$ (do not use the calculator).

**8.** (8pts) Below is the graph of the function $r = f(\theta)$ in rectangular $r$-$\theta$ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates, indicating corresponding parts of the graphs.




**9.** (10pts) If the long hand of the clock has length 5 in and the short hand 3 in, what is the distance between the tips of the hands at 7 o'clock?

**10.** (14pts) To determine distance $d$ to a building, sightings of points $A$ and $B$ on the building are made and they stand at angles of elevation $27°$ and $42°$. It is known that the distance from $A$ to $B$ is 11 meters.
a) Determine angles in the triangle $ABC$.
b) Find the distance to the building $d$.




**Bonus.** (10pts) Show that the area of a rectangle is one-half of product of lengths of diagonals times the sine of the angle between them.

**Trigonometry — Final Exam**
**MAT 145, Spring 2025— D. Ivanšić**

**Name:** _____

*Show all your work!*

| | |
|---|---|
| $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ | $\sin(2u) = 2 \sin u \cos u$ |
| $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ | $\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ |
| $\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ | $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$ |
| $\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ | $\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$ |

**1.** (12pts) If $\sin\theta = -\frac{2}{3}$ and $\theta$ is in the third quadrant, find the exact values of all the trigonometric functions of $\theta$. Draw a picture.

**2.** (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$\cos 45^\circ =$ $\qquad$ $\sin \frac{5\pi}{3} =$ $\qquad$ $\cot(540^\circ) =$ $\qquad$ $\csc \frac{11\pi}{6} =$

**3.** (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arcsin \frac{\sqrt{3}}{2} =$ $\qquad$ $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\qquad$ $\arcsin(-1.02) =$ $\qquad$ $\arctan\left(\frac{1}{\sqrt{3}}\right) =$

**4.** (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\tan(\arctan(-\sqrt{17})) =$ $\arcsin\left(\sin\dfrac{9\pi}{7}\right) =$

**5.** (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$22^\circ =$

$\dfrac{11\pi}{20}$ radians $=$

**6.** (10pts) You wrap a 2-foot string around a wheel with radius 17 inches. Consider the angle whose tip is at the center of the wheel and whose sides go through the ends of the string on the wheel. What is the size of the angle in radians? In degrees?

**7.** (8pts) Use an identity (sum, difference, half- or double-angle) to find the exact value of the trigonometric function below (do not use the calculator).

$\cos 105^\circ =$

**8.** (6pts) Show the identity.

$\sec\theta(\sec\theta - \cos\theta) = \tan^2\theta$

**9.** (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\cos^2\theta + 3\cos\theta + 1 = 0$

**10.** (14pts) Solve the triangle: $b = 5$, $a = 3$, $B = 37^\circ$

**11.** (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

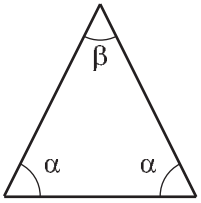
$(r, \theta) = \left(4, \frac{2\pi}{3}\right)$ $(r, \theta) = \left(-4, \frac{7\pi}{4}\right)$

**12.** (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct $\theta$. For each point, give three answers in polar coordinates, at least one of which has a negative $r$. Give exact answers — do not use the calculator.

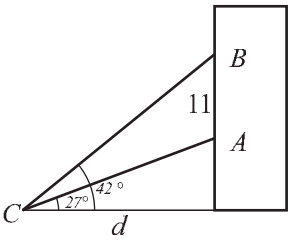
$(x, y) = (\sqrt{3}, 3)$ $(x, y) = (-2, -2)$

**13.** (10pts) A ski slope has angle $9^\circ$ with the horizontal. If a skier going straight down the hill travels 94 meters, what is the amount of her vertical descent?

**14.** (12pts) An isosceles triangle has two sides of equal length and two angles of same measure. If an isosceles triangle has angles $\alpha$, $\alpha$ and $\beta$, and $\cos\alpha = \frac{7}{9}$, find the exact value of $\cos\beta$ (do not use the calculator).




**15.** (14pts) To determine distance $d$ to a building, sightings of points $A$ and $B$ on the building are made and they stand at angles of elevation 27° and 42°. It is known that the distance from $A$ to $B$ is 11 meters.
a) Determine angles in the triangle $ABC$.
b) Find the distance to the building $d$.



**Bonus.** (10pts) Suppose that $\frac{3\pi}{2} < \alpha < 2\pi$ and $\frac{\pi}{2} < \beta < \pi$ are angles so that $\cos\alpha = \frac{1}{4}$ and $\sin\beta = \frac{2}{3}$. Find the exact value of $\sin(2\alpha + 2\beta)$.