$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v \qquad\qquad \sin(2u) = 2\sin u \cos u$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v \qquad\qquad \cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \qquad\qquad \tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

$$\cos^2 \frac{u}{2} = \frac{1 + \cos u}{2} \qquad \sin^2 \frac{u}{2} = \frac{1 - \cos u}{2} \qquad \tan^2 \frac{u}{2} = \frac{1 - \cos u}{1 + \cos u}$$

1. (12pts) If $\sin \theta = -\frac{5}{8}$ and θ is in the fourth quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

2. (12pts) Without using the calculator, find the exact values of the following trigonometric functions. Draw the unit circle and the appropriate angle to infer the values from the picture.

$$\cos 60° = \qquad\qquad \sin \frac{5\pi}{4} = \qquad\qquad \sec(-90°) = \qquad\qquad \tan \frac{8\pi}{3} =$$

3. (9pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$$\arccos \frac{\sqrt{3}}{2} = \qquad \arcsin\left(-\frac{\sqrt{2}}{2}\right) = \qquad \arccos(2) = \qquad \arctan(-\sqrt{3}) =$$

4. (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$$\sin(\arcsin 0.2) = \qquad\qquad \arccos\left(\cos\frac{8\pi}{5}\right) =$$

5. (6pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$63° =$

$\dfrac{7\pi}{15}$ radians $=$

6. (10pts) Apple's new headquarters building is in the shape of a ring with outer diameter 460 meters. If we refer to points on the circle via correspondence to a clock, how far would a person have to walk along the outside wall to get from a point at 1 o'clock to a point at 8 o'clock, going the long way?

7. (8pts) Use an identity (sum, difference, half- or double-angle) to find the exact value of the trigonometric function below (do not use the calculator).

$\cos 195° =$

8. (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos \theta = -0.25$

9. (14pts) Solve the equation in radians.
a) Give a general formula for all solutions.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2 \sin^2 \theta - \sin \theta - 1 = 0$

10. (14pts) Solve the triangle: $a = 7$, $c = 6$, $A = 38°$

11. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$$(r, \theta) = \left(3, \frac{\pi}{6}\right)$$

$$(r, \theta) = \left(-4, \frac{3\pi}{4}\right)$$

12. (10pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

$$(x, y) = (5\sqrt{3}, -5)$$

$$(x, y) = (-4, 4)$$

13. (10pts) A kite attached to a 60 ft string is flying so that the angle of elevation from the ground anchor to the kite is 49°. How high above the ground is the kite?

14. (11pts) To determine distances to a location C across the river, a surveyor puts poles at points A and B that are 32 meters apart. Using the poles, she is able to determine that the angle between lines of sight AB and AC from point A is 63° and the angle between lines of sight BA and BC from point B is 46°.
a) How far apart are A and C?
b) How far apart are B and C?

A
63°
C
32
46°
B
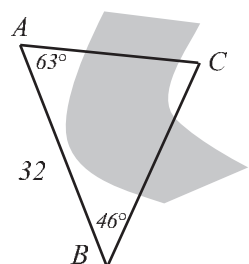
15. (13pts) Two planes leave an airport: one flies $N12°W$ at 250 mph, and the other flies $S37°E$ at 300 mph. What is the distance c between the planes after two hours?

N
12°
c
37°
S
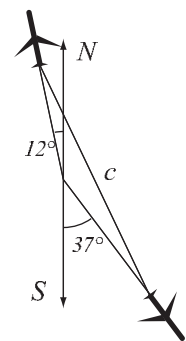
Bonus. (7pts) A circle of radius 16 meters is inscribed in a regular hexagon. Find the exact value of the perimeter of the hexagon (not a calculator approximation).
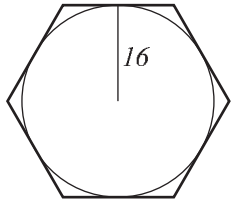
Bonus. (8pts) In a circle of radius a, the large triangle, whose bottom side is a diameter, is split into two triangles as shown.
a) Find the expression for the area of each of the two smaller triangles in terms of a and α.
b) Show the areas are equal.