College Algebra — Joysheet 11 MAT 140, Spring 2025 — D. Ivanšić

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Covers: 5.4-5.6 *Show all your work!*

1. (5pts) If $\log_a 4 = 0.6667$ and $\log_a 9 = 1.0566$, calculate:

$\log_a 36 = \log_a 4 + \log_a 9$
$= 0.6667 + 1.0566$
$= 1.7233$

$\log_a \frac{16}{9} = \log_a \frac{4^2}{9} = \log_a 4^2 - \log_a 9$
$= 2\log_a 4 - \log_a 9$
$= 2 \cdot 0.6667 - 1.0566$
$= 0.2768$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 (81x^4y^9) = \log_3 81 + \log_3 x^4 + \log_3 y^9$
$= 4 + 4\log_3 x + 9\log_3 y$

$\ln \frac{\sqrt{e}x^3y^5}{\sqrt[3]{x^7}z^4} = \ln\sqrt{e} + \ln x^3 + \ln y^5 - \ln \sqrt[3]{x^7} - \ln z^4$
$= \frac{1}{2} + 3\ln x + 5\ln y - \frac{7}{3}\ln x - 4\ln z$
$= \frac{1}{2} + \frac{2}{3}\ln x + 5\ln y - 4\ln z$

3. (12pts) Write as a single logarithm. Simplify if possible.

$3\log_2(4x^2) - \log_2(48y^5) + 3\log_2 x^4 = \log_2 (4x^2)^3 - \log_2(48y^5) + \log_2 (x^4)^3$
$= \log_2 \frac{(4x^2)^3(x^4)^3}{48y^5} = \log_2 \frac{\cancel{64}^{4}x^6x^{12}}{{}_{3}\cancel{48}y^5} = \log_2 \frac{4x^{18}}{y^5}$

$2\log_2(x^2+5x+4) - 3\log_2(x+4) - \log_2(x+1) =$
$= \log((x+4)(x+1))^2 - \log_2(x+4)^3 - \log_2(x+1)$
$= \log_2 \frac{\cancel{(x+4)^2}(x+1)^{\cancel{2}\,1}}{(x+4)^{\cancel{3}\,1}\cancel{(x+1)}} = \log_2 \frac{x+1}{x+4}$

4. (3pts) Simplify. $\log 10^{5a-2} = 5a-2$ $\quad 7^{\log_7 \sqrt{x+13}} = \sqrt{x+13}$

Solve the equations.

5. (5pts) $36^{x-3} = 6^{4x+3}$

$(6^2)^{x-3} = 6^{4x+3}$

$6^{2x-6} = 6^{4x+3}$

$2x-6 = 4x+3 \quad |-2x-3$

$-9 = 2x$

$x = -\frac{9}{2}$

6. (7pts) $5^{x+1} = 3^{2x-5} \quad | \ln$

$\ln 5^{x+1} = \ln 3^{2x-5}$

$(x+1)\ln 5 = (2x-5)\ln 3$

$\ln 5 x + \ln 5 = 2\ln 3 x - 5\ln 3$

$\ln 5 + 5\ln 3 = 2\ln 3 x - \ln 5 x$

$\ln 5 + 5\ln 3 = x(2\ln 3 - \ln 5)$

$x = \frac{\ln 5 + 5\ln 3}{2\ln 3 - \ln 5} = 12.083465$

7. (5pts) An investor puts \$15,000 into a stock. The stock's value increases by 7% every year, so after t years the value of the stock holding is given by the function $V(t) = 15 \cdot (1.07)^t$ (in thousands). When will the stock holding be worth \$40,000?

$V(t) = 15 \cdot 1.07^t$

$40 = 15 \cdot 1.07^t$

$1.07^t = \frac{40}{15}$

$1.07^t = \frac{8}{3} \quad | \ln$

$\ln 1.07^t = \ln \frac{8}{3}$

$t \ln 1.07 = \ln \frac{8}{3}$

$t = \frac{\ln \frac{8}{3}}{\ln 1.07} = 14.496731$

After about 14.5 years

8. (12pts) According to census data, Nashville, TN, had 546,000 inhabitants in 2000 and 689,000 in 2020. Assume the population of Nashville grows exponentially.
a) Write the function describing the number $P(t)$ of people in Nashville t years after 2000. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 800,000?

$P(t) = 546\, e^{kt}$, t = years since 2000

$689 = P(20) = 546\, e^{k \cdot 20}$

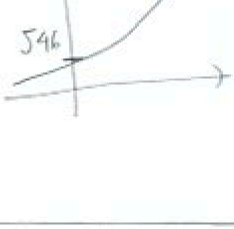
$\frac{689}{546} = e^{20k} \quad | \ln$

$\ln \frac{689}{546} = \ln e^{20k}$

$\ln \frac{689}{546} = 20k$

$k = \frac{\ln \frac{689}{546}}{20} = 0.0116311$

$P(t) = 546\, e^{0.0116311t}$



c) $800 = 546\, e^{0.011..t}$

$\frac{800}{546} = e^{0.011..t}$

$\ln \frac{800}{546} = 0.011..t$

$t = \frac{\ln \frac{800}{546}}{0.0116311} = 32.842316$

After about 33 years, so in 2033