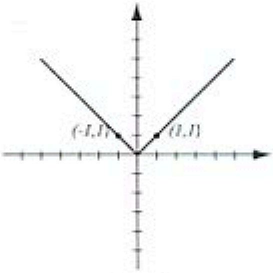
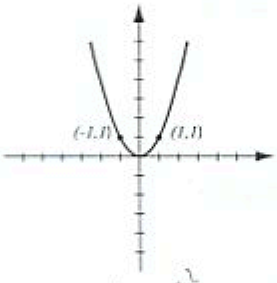
College Algebra — Final Exam **MAT 140C, Spring 2025 — D. Ivanšić**	Name: Saul Ocean *Show all your work!*

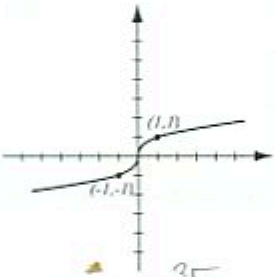
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



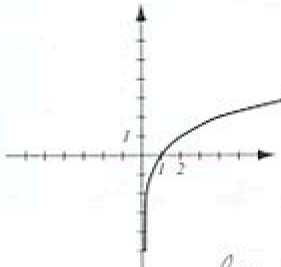
$y=|x|$



$y=x^2$



$y=\sqrt[3]{x}$



$y=\log_a x,\ a>1$

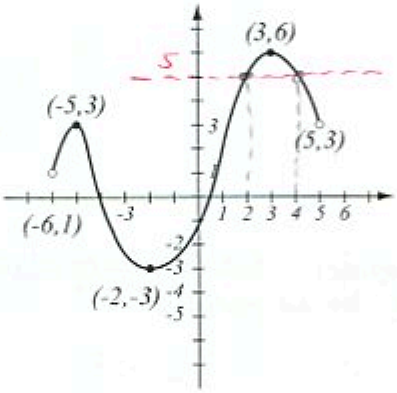
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(3)=$ 6 $\quad f(-7)=$ not defined

b) What is the domain of f? $(-6,5)$

c) What is the range of f? $[-3,6]$

d) What are the solutions of the equation $f(x)=5$? $x=2,4$



3. (11pts)

a) Find the equation of the line that passes through point $(4,1)$ and has y-intercept 2.

b) Find the equation of the line (in form $y=mx+b$) that is perpendicular to the line in a) and also passes through the point $(4,1)$.

c) Draw both lines.

a) line through $(4,1),\ (0,2)$

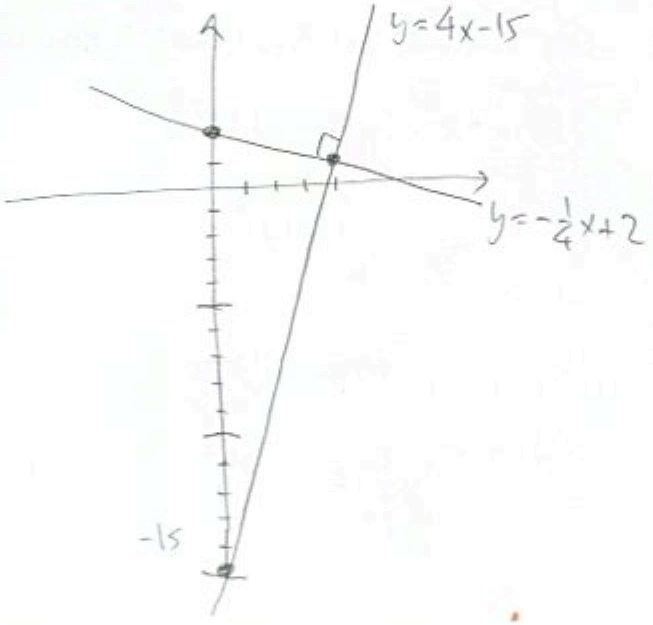
$m=\frac{2-1}{0-4}=\frac{1}{-4}=-\frac{1}{4} \qquad y=-\frac{1}{4}x+2$

b) slope of perp. line is $-\frac{1}{-\frac{1}{4}}=4$

$y-1=4(x-4)$

$y=4x-16+1$

$y=4x-15$



4. (3pts) Find the domain of the function $f(x) = \log_3(5 - 3x)$ and write it in interval notation.

Must have $5-3x>0$ $\quad (-\infty, \frac{5}{3})$

$5>3x$

$x<\frac{5}{3}$

5. (6pts) Solve and write the solution in interval notation.

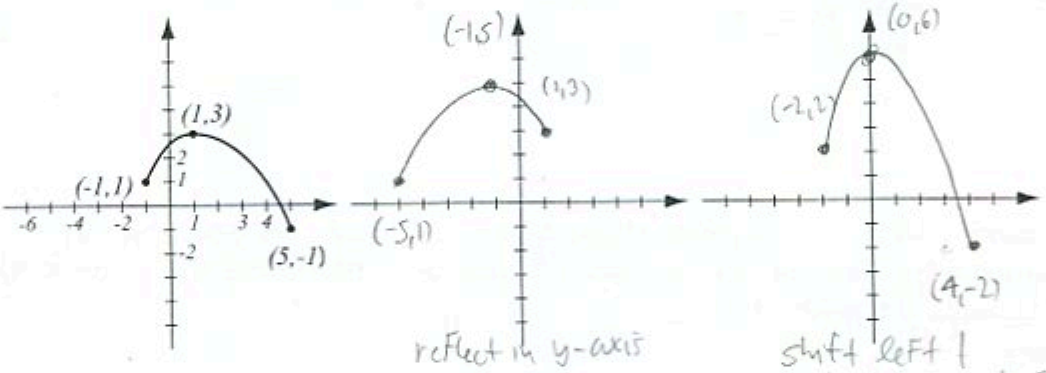
$|x - 3| < 7$

$-4 \quad 10$

$-7<x-3<7$

$-4<x<10$ $\quad (-4,10)$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) + 2$ and $2f(x + 1)$ and label all the relevant points.



reflect in y-axis
shift up 2

shift left 1
vert. stretch, Factor = 2

7. (5pts) Let $f(x) = 52e^{x+3}$. Find the formula for $f^{-1}(x)$.

$y = 52e^{x+3}$

$\ln| \quad \frac{y}{52} = e^{x+3}$

$\ln\frac{y}{52} = \ln e^{x+3}$

$\ln\frac{y}{52} = x+3$

$x = \ln\frac{y}{52} - 3$

$f^{-1}(y) = \ln\frac{y}{52} - 3$

8. (12pts) The quadratic function $f(x) = x^2 + 4x + 4$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int: $f(0) = 4$

x-int. $x^2 + 4x + 4 = 0$

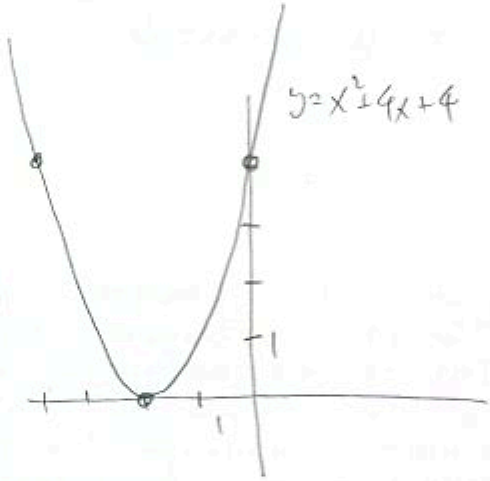
$(x+2)^2 = 0$

$x = -2$

b) $h = -\frac{b}{2a} = -\frac{4}{2\cdot 1} = -2$

$k = f(-2) = (-2)^2 + 4\cdot(-2) + 4 = 0$

(-4,4) by symmetry



9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_6\left(216x^5\sqrt[3]{y^2}\right) = \log_6 216 + \log_6 x^5 + \log_6 y^{\frac{2}{3}}$

$= 3 + 5\log_6 x + \frac{2}{3}\log_6 y$

10. (6pts) Write as a single logarithm. Simplify if possible.

$2\log(x-2) - \log(x^2 - 5x + 6) + 3\log(x-3) = \log(x-2)^2 - \log((x-3)(x-2)) + \log(x-3)^3$

$(x-3)(x-2)$

$= \log \frac{(x-2)^{\cancel{2}\,1}(x-3)^{\cancel{3}\,2}}{\cancel{(x-3)}\cancel{(x-2)}} = \log((x-2)(x-3)^2)$

11. (8pts) Let $f(x) = 3x + 5$, $g(x) = x^2 - 2$. Find the following (simplify where possible):

$(fg)(x) = (3x+5)(x^2-2)$

$= 3x^3 + 5x^2 - 6x - 10$

$(g \circ f)(x) = g(f(x)) = g(3x+5)$

$= (3x+5)^2 - 2 = (3x)^2 + 2 \cdot 3x \cdot 5 + 5^2 - 2$

$= 9x^2 + 30x + 23$

12. (20pts) The polynomial $P(x) = x^4 - 9x^2$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) like x^4 $\cup$

b) $x^4 - 9x^2 = x^2(x^2-9) = x^2(x-3)(x+3)$

zero	0	-3	3
mult	2	1	1

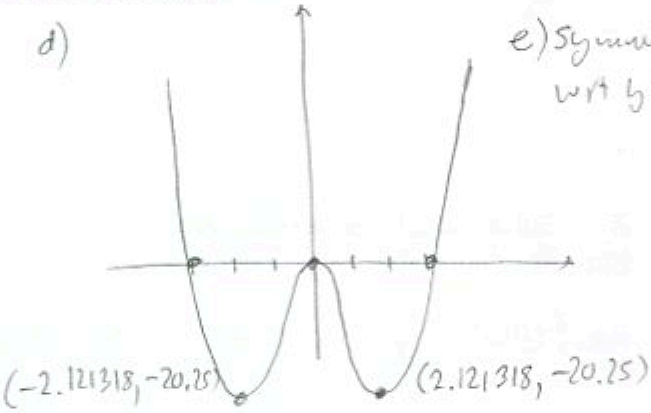
y-int. $P(0)=0$

c) $P(-x) = (-x)^4 - 9(-x)^2$

$= x^4 - 9x^2 = P(x)$

even

d)



e) symmetric w.r.t. y-axis

turning points: $(0,0)$,
$(2.121318, -20.25)$
$(-2.121318, -20.25)$

13. (8pts) Solve the equation.

$3 + \sqrt{3x-5} = x$ $\quad | -3$

$\sqrt{3x-5} = x - 3$ $\quad |^2$

$3x - 5 = x^2 - 2 \cdot x \cdot 3 + 9$

$x^2 - 6x + 9 = 3x - 5$ $\quad | -3x+5$

$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = 2$ or $x = 7$ only solution

Check:

$3 + \sqrt{3 \cdot 2 - 5} \stackrel{?}{=} 2$

$3 + \sqrt{1} = 2$ no

$3 + \sqrt{3 \cdot 7 - 5} = 7$

$3 + \sqrt{16} = 7$ yes

14. (14pts) You drive to Cadiz on the highway at 72mph. On the way back, you take the scenic road, driving 63mph. This road is 18 miles longer and takes a half hour longer to drive.
a) How long did it take you to drive to Cadiz?
b) How long is the scenic road?

dist, rate time

here $\xrightarrow{d,\ 72,\ t}$ Cadiz

$\xleftarrow{}$

$d+18,\ 63,\ t+\frac{1}{2}$ (30 min $= \frac{1}{2}$ hr)

$d = 72t$

$d+18 = 63(t+\frac{1}{2})$

$72t+18 = 63(t+\frac{1}{2})$

$72t+18 = 63t+31.5 \quad |-63t-18$

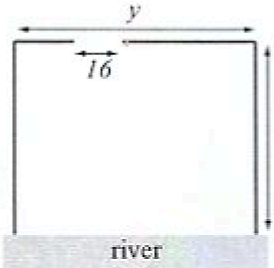
$9t = 13.5$

$t = \frac{13.5}{9} = 1.5$ hrs

a) $t = 1.5$ hrs

b) $63\cdot(1.5+\frac{1}{2}) = 63\cdot 2 = 126$ miles

15. (14pts) Farmer Felix is constructing a rectangular enclosure in a field along a river. He has 800 feet of fencing. The side along the river does not need fencing, and the enclosure has one 16-foot opening. Felix's goal is to maximize the area of the enclosure.
a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the enclosure that has the biggest possible area and what is the biggest possible area?

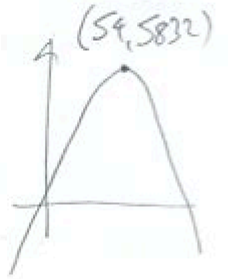


a) $2x+y-16 = 800 \Rightarrow 2x+y = 816 \Rightarrow y = 816-2x$

$A = x\cdot y = x(816-2x) = -2x^2+816x$

a) Domain:
must have $x \geq 0$
$y \geq 16$
$816-2x \geq 16$
$800 \geq 2x$
$x \leq 400$
$[0, 400]$

b)



$h = -\frac{816}{2(-2)} = \frac{816}{4} = 204$

$A(54) = -2\cdot 54^2 + 816\cdot 54 = 83{,}232$

$816-2\cdot 204$

dimensions: 204×408 ft

max area: ft^2

16. (12pts) Census data has the population of Knoxville, TN, as 165,000 in 1990 and 191,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 1990. Graph it on paper.
b) Find the predicted population in the year 2030.

a) $P(t) = 165 e^{kt}$ t - years since 1990

$191 = P(30) = 165 e^{k \cdot 30}$

$191 = 165 e^{k \cdot 30}$

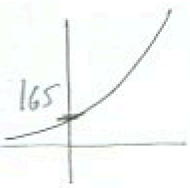
$\frac{191}{165} = e^{30k}$ | ln

$\ln \frac{191}{165} = \ln e^{30k}$

$\ln \frac{191}{165} = 30k$

$k = \frac{\ln \frac{191}{165}}{30} = 0.0048776$

$P(t) = 165 e^{0.0048776 t}$



b) 2030 is 40 years after 1990

$P(40) = 165 e^{0.0048776 \cdot 40}$

$= 200.547156$

In 2030, Knoxville is predicted to have 200,547 inhabitants

Bonus (10pts) Find the equation of a parabola whose vertex is $(2, -5)$ and whose y-intercept is 3. One way to approach this is to write $y = ax^2 + bx + c$ and find a, b and c based on the information above.

$y = ax^2 + bx + c$

when $x = 0$, $y = 3$

$3 = 0 + 0 + c$

$c = 3$

$2 = -\frac{b}{2a}$ so $b = -4a$

$(2, -5)$ is on parabola:

$-5 = a \cdot 2^2 + b \cdot 2 + 3$

$4a + 2b + 3 = -5$

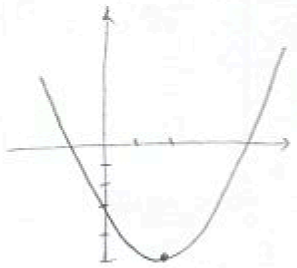
$4a + 2(-4a) = -8$

$-4a = -8$

$a = 2$

$b = -4 \cdot 2 = -8$

$y = 2x^2 - 8x + 3$



$(2, -5)$