College Algebra — Exam 3	Name: Saul Ocen
MAT 140C, Spring 2025 — D. Ivanšić	*Show all your work!*

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $3i(7+2i)^2 = 3i\left(7^2 + 2\cdot7\cdot2i + (2i)^2\right) = 3i(49 + 28i + 4i^2)$

$= 3i(45 + 28i) = 135i + 84i^2 = -84 + 135i$

2. (6pts) $\frac{1-5i}{7-3i} \cdot \frac{7+3i}{7+3i} = \frac{7+3i-35i-15i^2}{7^2 - 9i^2} = \frac{7-32i+15}{49+9} = \frac{22-32i}{58}$

$= \frac{11-16i}{29}$

3. (4pts) Simplify and justify your answer.

$i^{135} = i^{132} \cdot i^3 = 1 \cdot i \cdot i \cdot i = -i$

$132 = 4 \cdot 34$

4. (6pts) Solve the equation by completing the square.

$x^2 - 12x = 12$ $\quad 1 + 6^2$

$x^2 - 2\cdot x \cdot 6 + 6^2 = 12 + 6^2$

$(x-6)^2 = 48$

$x - 6 = \pm\sqrt{48}$ $\quad 16\cdot3$

$x = 6 \pm 4\sqrt{3}$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|3x - 7| < 4$

$-4 < 3x - 7 < 4$

$3 < 3x < 11$

$1 < x < \frac{11}{3}$

$1 \quad \frac{11}{3}$

$\left(1, \frac{11}{3}\right)$

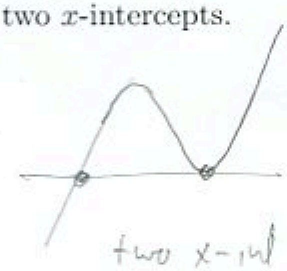
6. (6pts) Let $P(x)$ be a polynomial of degree 3.

a) Draw a graph of P that has exactly one x-intercept and no turning points.

b) Draw a graph of P that has exactly two x-intercepts.

a)

b)



7. (12pts) The quadratic function $f(x) = -x^2 - 2x + 3$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) x-int:

$-x^2-2x+3=0$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

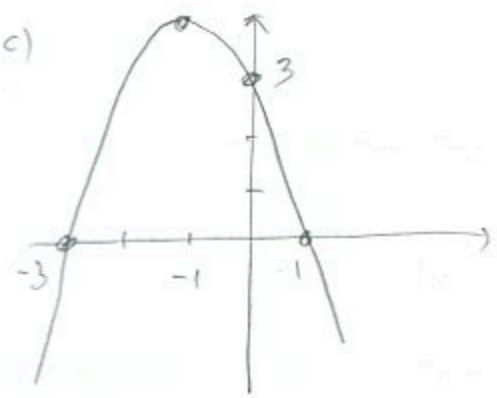
$x=-3, 1$

$f(0)=3$ · y-int

b) $h=-\frac{-2}{2\cdot(-1)}=-1$

$k=f(-1)=-(-1)^2-2(-1)+3=-1+2+3=4$

c)



Solve the equations:

8. (8pts) $\frac{x}{x+4} = \frac{2}{x+1} + \frac{5x-1}{x^2+5x+4}$ $\quad (x+4)(x+1)$

$\frac{x}{x+4}(x+4)(x+1) = \frac{2}{x+1}(x+4)(x+1) + \frac{5x-1}{(x+4)(x+1)}(x+4)(x+1)$

$x^2+x = 2x+8+5x-1$

$x^2+x = 7x+7 \quad |-7x-7$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x=7, (-1)$ ← gives 0 in denom.

$x=7$ only sol.

9. (8pts) $\sqrt{3x+7}-x=1$

$\sqrt{3x+7}=1+x \quad |^2$

$3x+7=1^2+2\cdot1\cdot x+x^2$

$x^2+2x+1=3x+7 \quad |-3x-7$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3, -2$

Check $\sqrt{3\cdot3+7}-3\overset{?}{=}1$

$\sqrt{16}-3=1$ yes

$\sqrt{3\cdot(-2)+7}-(-2)\overset{?}{=}1$

$\sqrt{1}+2=1$ no

$x=3$ only solution

10. (14pts) The polynomial $f(x) = (x-2)^2(x+4)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

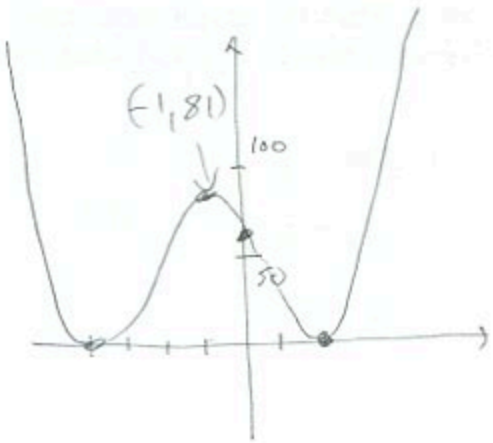
a) $(x)^2(x)^2 = x^4$

like x^4, ∪

b)

zero	2	−4
mult	2	2

$f(0) = (-2)^2 \cdot 4^2 = 64$



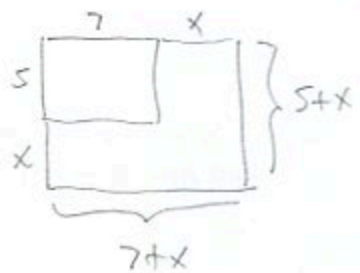
d) turning points:

(-4,0)

(2,0)

(-1,81)

11. (12pts) Starting with a 5 ft×7 ft rectangle, we increased the width and length by the same amount to get a rectangle with area 50 ft^2. How much was added to the width and length of the 5 × 7 rectangle?



$(5+x)(7+x) = 50$

$35 + 12x + x^2 = 50 \quad |-50$

$x^2 + 12x - 15 = 0$

$144 + 60 = 204$

$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 1 \cdot (-15)}}{2 \cdot 1} = \frac{-12 \pm \sqrt{204}}{2} = \frac{-12 \pm 2\sqrt{51}}{2}$

4.51

$= \frac{2(-6 \pm \sqrt{15})}{2} = -6 \pm \sqrt{51}$

$x = -6 + \sqrt{51}$ is a soln.

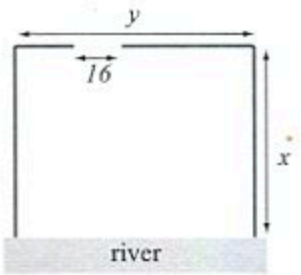
$= 1.141428$

$-6 - \sqrt{51} < 0$

↑

not a soln.

12. (14pts) Farmer Felix is constructing a rectangular enclosure in a field along a river. He has 1000 feet of fencing. The side along the river does not need fencing, and the enclosure has one 16-foot opening. Felix's goal is to maximize the area of the enclosure.
a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the enclosure that has the biggest possible area and what is the biggest possible area?



a) $A = xy = x(1016 - 2x) = -2x^2 + 1016x$

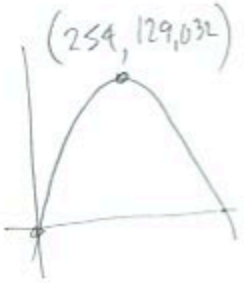
$2x + y - 16 = 1000$

$2x + y = 1016$

$y = 1016 - 2x$

Domain:
Must have
$x \geq 0$
$y \geq 0$
$1016 - 2x \geq 0$
$2x \leq 1016$
$x \leq 508$

b)



$h = -\frac{b}{2a} = -\frac{1016}{2 \cdot (-2)} = 254$

$k = -2 \cdot 254^2 + 106 \cdot 254 = 129{,}032 \text{ Ft}^2$

$y = 1016 - 2 \cdot 254$

Dimensions: 254 by 508

Max area: $129{,}032 \text{ ft}^2$

Bonus. (10pts) Find the equation of a parabola whose vertex is $(2, -5)$ and whose y-intercept is 3. One way to approach this is to write $y = ax^2 + bx + c$ and find a, b and c based on the information above.

$y = ax^2 + bx + c$

When $x = 0$, $y = 3$

$3 = 0 + 0 + c$

$c = 3$

$-\frac{b}{2a} = 2 \Rightarrow b = -4a$, put in 2nd eq.

When $x = 2$, $y = -5$

$-5 = a \cdot 2^2 + b \cdot 2 + 3$

$4a + 2b = -8$

$4a + 2(-4a) = -8$

$-4a = -8$

$a = 2$

$b = -8$

$y = 2x^2 - 8x + 3$