College Algebra — Exam 2
MAT 140C, Spring 2025 — D. Ivanšić

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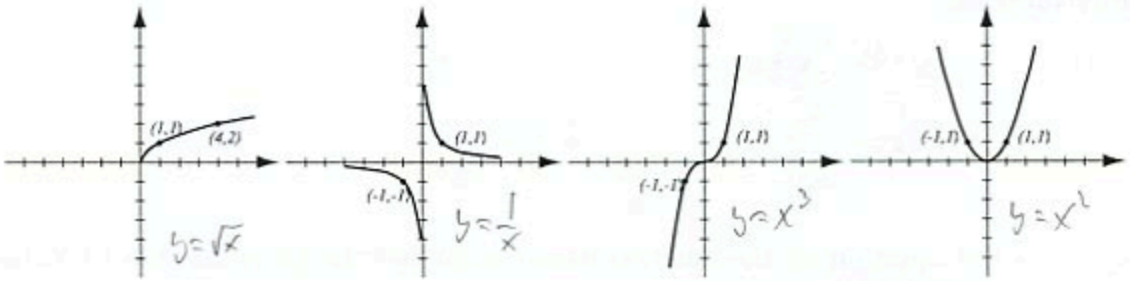
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = \sqrt{x}$ $y = \frac{1}{x}$ $y = x^3$ $y = x^2$

2. (20pts) Let $f(x) = \dfrac{2x}{x-3}$, $g(x) = \dfrac{1}{x}$.

Find the following (simplify where possible):

$(f-g)(2) = f(2) - g(2) = \frac{2\cdot 2}{2-3} - \frac{1}{2}$

$= \frac{4}{-1} - \frac{1}{2} = -4 - \frac{1}{2} = -\frac{9}{2}$

$(fg)(5) = f(5)\cdot g(5) = \frac{2\cdot 5}{5-3} \cdot \frac{1}{5}$

$= \frac{10}{2} \cdot \frac{1}{5} = 5 \cdot \frac{1}{5} = 1$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\frac{2x}{x-3}}{\frac{1}{x}} = \frac{2x}{x-3} \cdot \frac{x}{1} = \frac{2x^2}{x-3}$

$(g \circ f)(4) = g(f(4)) = g\left(\frac{2\cdot 4}{4-3}\right) = g\left(\frac{8}{1}\right)$

$= g(8) = \frac{1}{8}$

$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{2\cdot\frac{1}{x}}{\frac{1}{x} - 3} \cdot \frac{x}{x} = \frac{2}{1-3x}$

The domain of $f - g$ in interval notation

Domain f: can't have $x - 3 = 0$, $x = 3$

Domain g: can't have $x = 0$

f ~~all numbers~~

g ~~all numbers~~

$f-g$ ~~all numbers~~ 0 3

Domain $f-g$: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

3. (6pts) Consider the function $h(x) = \dfrac{7}{3x^2+1}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = 3x^2+1$ $\qquad$ $g(x) = 3x^2$

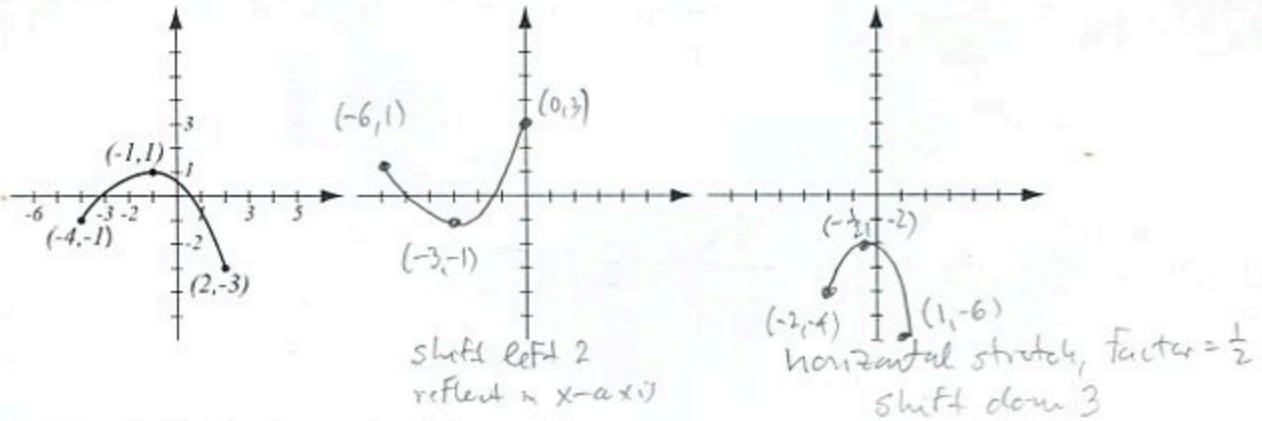
$f(x) = \dfrac{7}{x}$ $\qquad$ $f(x) = \dfrac{7}{x+1}$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of x^3, reflected over the y-axis.
b) shape of $y = x^2$, stretched vertically by factor 2 and then shifted up 1.

$x^3 \rightsquigarrow (-x)^3$

$x^2 \rightsquigarrow 2x^2 \rightsquigarrow 2x^2+1$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+2)$ and $f(2x)-3$ and label all the relevant points.

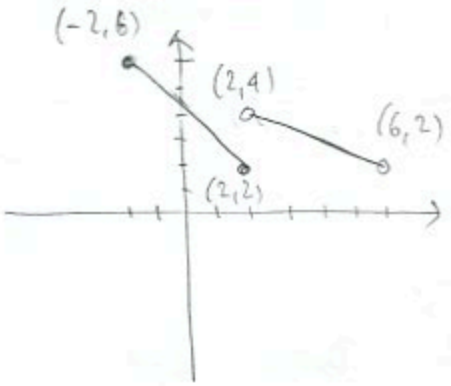


6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 4-x, & \text{if } -2 \le x \le 2 \\ 5-\frac{1}{2}x, & \text{if } 2 < x < 6 \end{cases}$$

x	$4-x$
-2	6
2	2

x	$5-\frac{1}{2}x$
2	4
6	2



7. (5pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} x^2 + 2x - 3, & \text{if } x < -3 \\ \sqrt{2x+7}, & \text{if } -3 \le x \le 2 \end{cases}$$

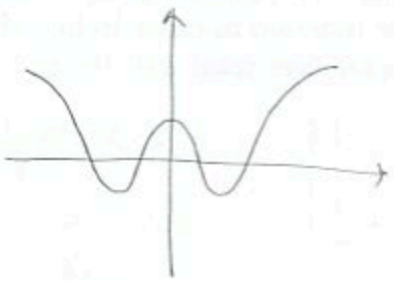
$f(0) = \sqrt{2 \cdot 0 + 7} = \sqrt{7}$

$f(7) =$ not defined

$f(-6) = (-6)^2 + 2(-6) - 3 = 36 - 12 - 3$
$= 21$

8. (3pts) Sketch a graph of an even function. You can draw any curve you like, as long as it has the property requested.

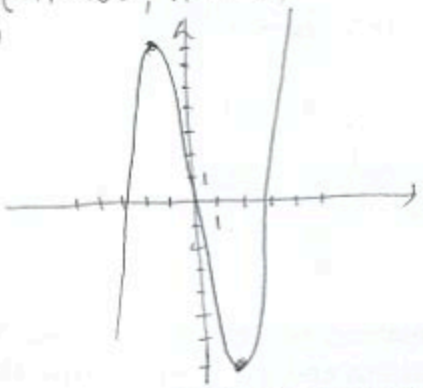
Need graph symmetric wrt y-axis



9. (20pts) Let $f(x) = x^3 - 7x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

(-1,527524, 7,128451)

a)



(1,527524, -7.128451)

d) local max: $7,128451 = f(-1.527524)$

local min: $-7.128451 = f(1.527524)$

e) Increasing on $(-\infty, -1,527524)$ and $(1,527524, \infty)$

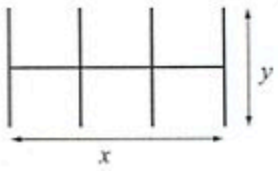
Decreasing on $(-1,527524, 1.527524)$

b) $f(-x) = (-x)^3 - 7(-x)$
$= -x^3 + 7x = -f(x)$

Function is odd

c) graph is symmetric wrt origin

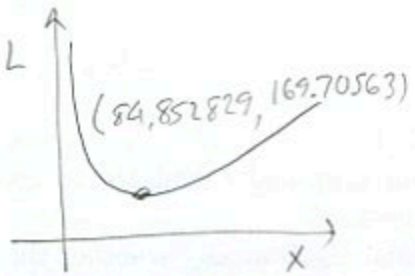
10. (14pts) Entrepreneur Kaja is building a block of six self-storage units, with total area 1800 square feet. They are open on one side to accommodate a garage door. Kaja's goal is to minimize building cost, which is same as minimizing the total length of the walls.
a) Express the total length of walls of the block as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block that has the smallest total wall length? What is the smallest total wall length?



$L = x + 4y = x + 4 \cdot \frac{1800}{x} = x + \frac{7200}{x}$

$1800 = xy$

$y = \frac{1800}{x}$



Domain:

Must have $x > 0$

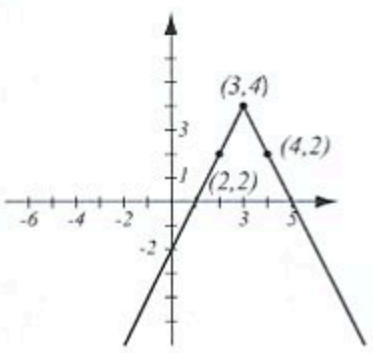
$(0, \infty)$

Dimensions: 84.852829 by 21.2132

$\frac{1800}{84.852829}$

Minimal wall length: 169.70563

Bonus. (10pts) The graph below was obtained by transformations of a graph of a standard function. Identify the standard function and the transformations and use them to write the formula for the function in the picture.



Looks like $y = |x|$ was transformed