Calculus 1 — Exam 5
MAT 250, Spring 2024 — D. Ivanšić

Name: ____________________

Show all your work!

Find the following antiderivatives or definite integrals.

1. (3pts) $\displaystyle\int \frac{1}{\sqrt[4]{x}}\,dx =$

2. (3pts) $\displaystyle\int \sin(2x-\pi)\,dx =$

3. (6pts) $\displaystyle\int (u^2 - 3\sqrt{u})u^3\,du =$

4. (5pts) $\displaystyle\int_0^{\frac{\pi}{4}} 3\sec^2\theta\,d\theta =$

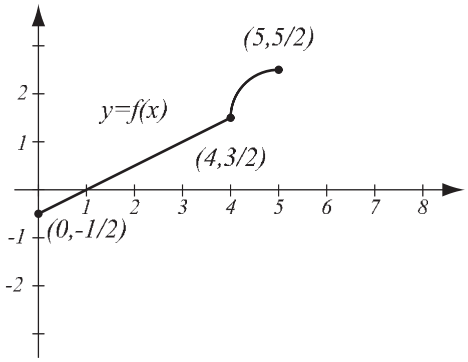
5. (6pts) $\displaystyle\int_{\sqrt{e}}^{e} x - \frac{1}{x}\,dx =$

6. (6pts) Find $f(x)$ if $f'(x) = e^x - \cos x$ and $f(0) = 4$.

7. (15pts) The function $f(x) = x^2 - 2$ is given on the interval $[0, 3]$.
a) Write the Riemann sum M_6 for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

8. (13pts) Find $\int_{-2}^{2} 2x - 2\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (10pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.



$\int_0^4 f(x)\,dx =$

$\int_4^5 f(x)\,dx =$

$\int_0^5 f(x)\,dx =$

10. (16pts) Consider the integral $\displaystyle\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \sin x\,dx$.

a) Use the inequality $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$, where $m \le f(x) \le M$ on $[a,b]$, to give an estimate of the integral. (A graph of $\sin x$ will help you find m and M.)

b) Evaluate the integral and verify your estimate from a).

11. (7pts) Write using sigma notation:

$$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} + \cdots + \frac{13}{256} =$$

12. (10pts) The rate at which temperature in an oven is changing is $\sqrt{t} + 2$ degrees Fahrenheit per minute.
a) Use the Net Change Theorem to find how much temperature changed from $t = 4$ to $t = 9$ minutes.
b) If at time $t = 4$ minutes the temperature in the oven was 180°F, what is the temperature at $t = 9$ minutes?

Bonus. (10pts) Show that $\displaystyle\sum_{i=1}^{n}(2i-1) = n^2$. (This is $1 + 3 + 5 + \cdots + (2n-1) = n^2$.)
Use either a picture with beads (what is a good way to picture n^2 beads?) that is cleverly divided up, or show it algebraically, for which you may find the formula $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ useful.