1. (32pts) Let $f(x) = \dfrac{x}{x^2 + 4}$. The domain of this function is all real numbers (you do not have to verify this). Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.

d) Use information from a)–c) to sketch the graph.

2. (18pts) Let $f(x) = \sin^2 \theta - \cos \theta$. Find the absolute minimum and maximum values of f on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

3. (14pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.
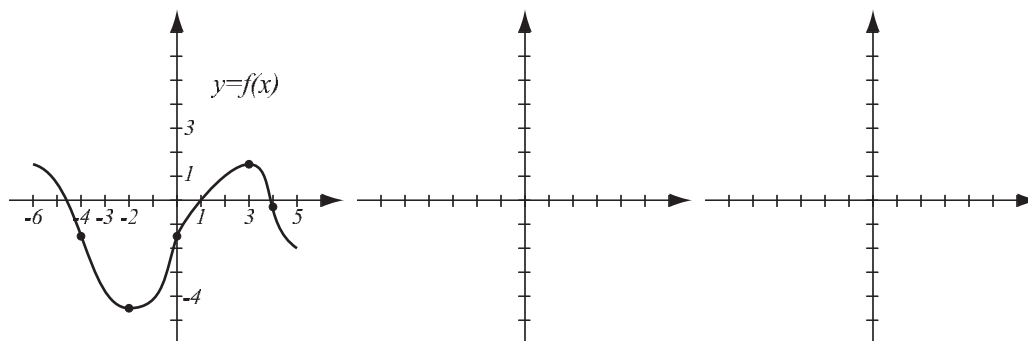
4. (14pts) Consider $f(x) = x^3 - 4x^2$ on the interval $[0, 2]$.

a) Verify that the function satisfies the assumptions of the Mean Value Theorem.

b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. (22pts) Among all rectangles with area 24, find the one with the smallest perimeter.

Bonus. (10pts) Draw the graph of a function that is defined for all real numbers that satisfies:

$f(-1) = -3$, $f(2) = 1$
$f'(x) > 0$ for all x in $(-1, 2)$
$f'(x) < 0$ for all x in $(-\infty, -1)$ and $(2, \infty)$
$f'(-1) = 0$, $f'(2)$ does not exist
$f''(x) > 0$ for all x in $(-\infty, 2)$ and $(2, 3)$
$f''(x) < 0$ for all x in $(3, \infty)$