Calculus 1 — Exam 3
MAT 250, Spring 2024 — D. Ivanšić

Name: _____

Show all your work!

Differentiate and simplify where appropriate:

1. (4pts) $\dfrac{d}{dx}\, x^2 5^x =$

2. (6pts) $\dfrac{d}{d\theta}\, e^\theta \sin(2\theta) =$

3. (7pts) $\dfrac{d}{du}\, \dfrac{e^u - \sin u}{e^u + \sin u} =$

4. (7pts) $\dfrac{d}{dx}\, \ln \dfrac{\sin^2 x}{\tan x} =$

5. (7pts) $\dfrac{d}{dv}\, \arcsin \sqrt{1 - v^2} =$

6. (9pts) Use logarithmic differentiation to find the derivative of $y = (\cos x)^{\cos x}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim_{x\to\infty} e^{-0.5x} =$

8. (7pts) $\lim_{x\to\infty} (\ln(x+2) - \ln(x^2-1)) =$

9. (7pts) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2} =$

10. (9pts) $\lim_{x\to\infty} x^2 \sin\dfrac{1}{x^2} =$

11. (8pts) $\lim_{x\to\infty} (x^4+3)^{\frac{1}{2x}} =$

12. (11pts) Let $f(x) = \arctan x$.
a) Write the linearization of $f(x)$ at $a = 0$.
b) Use the linearization to estimate $\arctan(-\frac{1}{2})$.
c) In the same coordinate system, draw graphs of the function and the linearization and determine if the estimate in b) is an overestimate or underestimate of $\arctan(-\frac{1}{2})$.

13. (9pts) A cube is measured to have side length of 3 meters, with maximum error 0.5 centimeters. Use differentials to estimate the maximum possible error when computing the surface area of the cube.

14. (7pts) Let $f(x) = x - \sqrt[3]{x}$. Use the theorem on derivatives of inverses to find $(f^{-1})'(6)$.

Bonus. (10pts) Find the limit.

$$\lim_{x \to 0+} \ln x \ln(x+1) =$$