Calculus 1 — Exam 2
MAT 250, Spring 2024 — D. Ivanšić

Name: _____

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(3x^6 - \dfrac{4}{x^4} + \dfrac{5}{\sqrt[6]{x}} + \pi^3\right) =$

2. (5pts) $\dfrac{d}{dx}\,(x^2+1)\cos x =$

3. (6pts) $\dfrac{d}{du}\,\dfrac{(u+1)^2}{(u-4)^3} =$

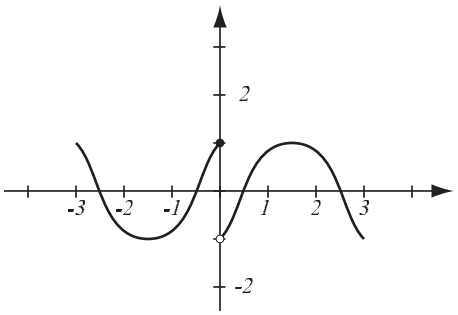
4. (6pts) $\dfrac{d}{d\theta}\,\dfrac{\cos\theta}{\cos\theta - \sin\theta} =$

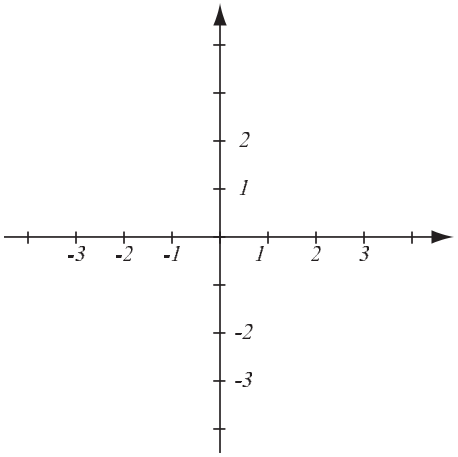
5. (6pts) $\dfrac{d}{dz}\,\tan\sqrt{\sec z} =$

6. (7pts) Let $y(x) = x^4$.
a) Write the first four derivatives of y.
b) What is the n-th derivative of y for $n \geq 5$?

7. (10pts) The graph of the function $f(x)$ is shown at right.

a) Where is $f(x)$ not differentiable? Why?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.





8. (12pts) Let $f(x) = 2x^2 - 5x + 1$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(2, -1)$.

9. (11pts) Let $g(x) = f(x^3)$ and $h(x) = \dfrac{(f(x))^2}{x}$.

a) Find the general expressions for $g'(x)$ and $h'(x)$.

b) Use the table of values at right to find $g'(1)$ and $h'(3)$.

x	1	2	3	4
$f(x)$	3	2	5	-3
$f'(x)$	2	-1	-4	2

10. (6pts) An ball thrown upwards has position (in feet, t in seconds) given by the formula $s(t) = -16t^2 + 40t$.

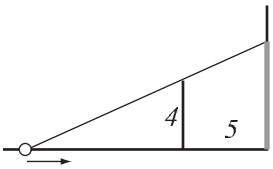
a) Write the formula for the velocity of the ball at time t.

b) What is the highest altitude that the ball reaches?

11. (11pts) Use implicit differentiation to find y' in general

$\sin(xy) = \sin(x^2) + \sin(y^2)$

12. (14pts) A light source is approaching a 4-meter pole that stands 5 meters in front of a tall wall. If the light source is moving at rate 0.5 meters per second when it is 3 meters from the pole, how fast is the shadow of the pole on the wall growing at that moment? *Hint: similar triangles.*



Bonus. (10pts) Find the points on the curve $y = x^2$ at which the tangent line passes through the point $(2, -1)$, which is not on the curve. *Hint: look for a point* (a, a^2) *on the curve so that the slope of the line through* (a, a^2) *and* $(2, -1)$ *is equal to the slope of the tangent line at* (a, a^2).