

Calculus 1 — Exam 1
MAT 250, Spring 2024 — D. Ivanšić

Name: _____

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$$\lim_{x \rightarrow -4} f(x) =$$

$$\lim_{x \rightarrow 2^-} f(x) =$$

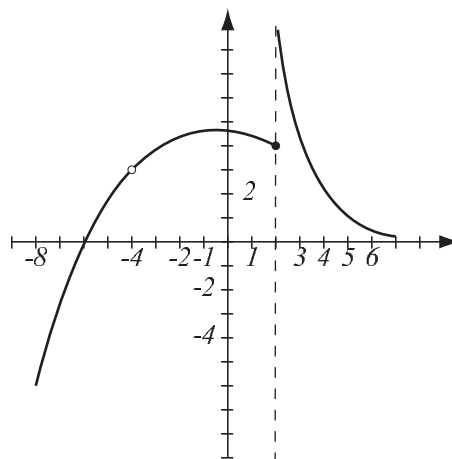
$$\lim_{x \rightarrow 2^+} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

List points in $(-\infty, \infty)$ where f is not continuous and justify why it is not continuous at those points.



2. (6pts) Let $f(x) = \frac{x^2 + 3}{x + 1}$.

a) State the domain of f .

b) Briefly explain why f is continuous on its domain.

3. (10pts) Find $\lim_{x \rightarrow 0} x^2 \left(3 \cos \frac{1}{x} + 5 \right)$. Use the theorem that rhymes with what a forest consists of.

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\lim_{x \rightarrow -2} \frac{\sqrt{x+3} - 1}{x+2} =$

5. (7pts) $\lim_{x \rightarrow \infty} \frac{x^2 + 3}{3x^2 - 5x + 7} =$

6. (5pts) $\lim_{x \rightarrow 3} \frac{x-3}{x^2 + 4x - 21} =$

7. (6pts) $\lim_{x \rightarrow 2^-} \frac{x^2 + 1}{2x - 4} =$

8. (7pts) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(2x)} =$

9. (14pts) The equation $x = \cos x$ is given.

- a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, \frac{\pi}{2})$.
- b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

10. (10pts) Consider the limit $\lim_{x \rightarrow 0} \frac{4^x - 1}{x}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values (no more than 5 per table) that will support your answer.

x	$\frac{4^x - 1}{x}$	x	$\frac{4^x - 1}{x}$

11. (12pts) Consider the function defined below. Is the function continuous at point $x = 0$?

$$f(x) = \begin{cases} \frac{(1+x)^2 - 1}{x} & \text{if } x < 0 \\ \frac{\sin(2x)}{x} & \text{if } x > 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Bonus. (10pts) For each group of properties, draw the graph of the function defined on $[1, 5]$ that satisfies them, if possible. Among the three, one is not possible — explain why.

$f(1) = 4, f(5) = 2$
graph of f does not
cross line $y = 6$
 f is continuous

$f(1) = 4, f(5) = 2$
graph of f does not
cross line $y = 3$
 f is continuous

$f(1) = 4, f(5) = 2$
graph of f does not
cross line $y = 3$