**Calculus 1 — Exam 5**
**MAT 250, Fall 2025 — D. Ivanšić**

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*Show all your work!*

Find the following antiderivatives or definite integrals.

**1.** (3pts) $\int \frac{1}{x^4}\,dx =$ $\int x^{-4}\,dx = \frac{x^{-3}}{-3} = -\frac{1}{3x^3} + C$

**2.** (3pts) $\int \sec^2(5\theta+\pi)\,d\theta =$ $\frac{\tan(5\theta+\pi)}{5} + C$

**3.** (6pts) $\int \frac{v^2+3v}{\sqrt{v}}\,dv =$ $\int \frac{v^2+3v}{v^{1/2}} = \int v^{\frac{3}{2}} + 3v^{\frac{1}{2}}\,dv = \frac{2}{5}v^{\frac{5}{2}} + 3\cdot\frac{2}{3}v^{\frac{3}{2}}$

$= \frac{2}{5}v^{\frac{5}{2}} + 2v^{\frac{3}{2}} + C$

**4.** (5pts) $\int_{\frac{\pi}{4}}^{\frac{2\pi}{3}} 4\sin\theta\,d\theta =$ $4(-\cos\theta)\Big|_{\pi/4}^{\frac{2\pi}{3}} = -4\left(\cos\frac{2\pi}{3} - \cos\frac{\pi}{4}\right)$

$= -4\left(-\frac{1}{2} - \frac{\sqrt{2}}{2}\right) = 4\cdot\frac{1+\sqrt{2}}{2} = 2(1+\sqrt{2})$

**5.** (6pts) $\int_1^2 x^2 - 3x + 2\,dx =$ $\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)\Big|_1^2 = \frac{1}{3}(2^3-1^3) - \frac{3}{2}(2^2-1^2) + 2(2-1)$

$= \frac{7}{3} - \frac{3}{2}\cdot 3 + 2 = \frac{7}{3} - \frac{9}{2} + 2 = \frac{14-27+12}{6} = -\frac{1}{6}$

**6.** (6pts) Find $f(x)$ if $f'(x) = e^{2x} - \frac{1}{x}$ and $f(1) = 5$.

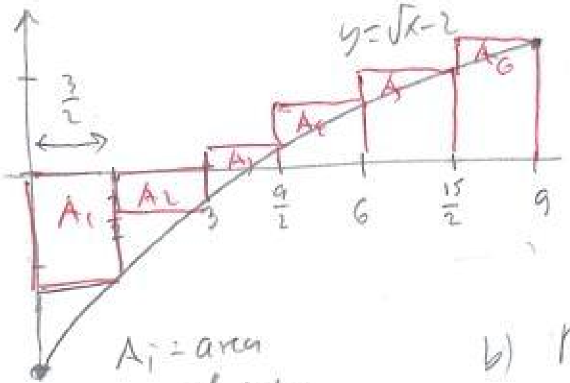
$f(x) = \frac{e^{2x}}{2} - \ln|x| + C$

$5 = f(1) = \frac{e^2}{2} - \underbrace{\ln|1|}_{=0} + C$

$5 = \frac{e^2}{2} + C$ $\quad C = 5 - \frac{e^2}{2}$

$f(x) = \frac{e^{2x}}{2} - \ln|x| + 5 - \frac{e^2}{2}$

**7.** (15pts) The function $f(x) = \sqrt{x} - 2$ is given on the interval $[0, 9]$.
a) Write the Riemann sum $R_6$ for this function with six subintervals, taking sample points to be end points. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does $R_6$ represent?



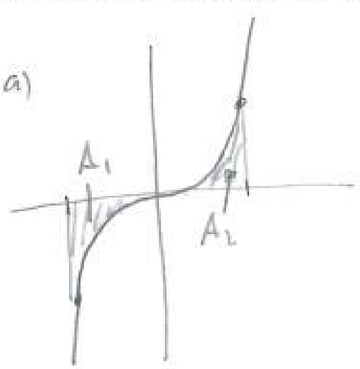

$A_i$ = area of i-th rectangle

a) $R_6 = \frac{3}{2}\left(\sqrt{\frac{3}{2}}-2 + \sqrt{3}-2 + \sqrt{\frac{9}{2}}-2 + \sqrt{6}-2 + \sqrt{\frac{15}{2}}-2 + \sqrt{9}-2\right)$

b) $R_6 = -A_1 - A_2 + A_3 + A_4 + A_5 + A_6$

**8.** (13pts) Find $\int_{-1}^{1} x^3\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture. You do not need to know the actual areas involved to find the integral: use symmetry.
b) Using the Evaluation Theorem.

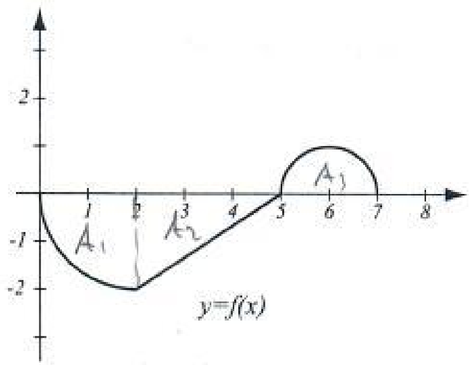
a)




$\int_{-1}^{1} x^3\,dx = -A_1 + A_2 = 0$ since $A_1 = A_2$ due to symmetry

b) $\int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Big|_{-1}^{1} = \frac{1}{4}(1^4 - (-1)^4) = 0$

**9.** (10pts) The graph of a function $f$, consisting of lines and parts of circles, is shown. Evaluate the integrals.




$\int_2^5 f(x)\,dx = -A_2 = -\frac{1}{2}\cdot 3\cdot 2 = -3$

$\int_5^7 f(x)\,dx = A_3 = \frac{1}{2}\cdot\pi\cdot 1^2 = \frac{\pi}{2}$

$\int_0^7 f(x)\,dx = -A_1 - A_2 + A_3 = -\frac{1}{4}\pi\cdot 2^2 - 3 + \frac{\pi}{2}$

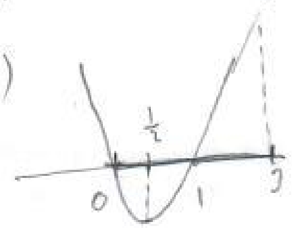
$= -\pi - 3 + \frac{\pi}{2} = -3 - \frac{\pi}{2}$

**10.** (16pts) Consider the integral $\displaystyle\int_0^3 x^2 - x\,dx$.

a) Use the inequality $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$, where $m \le f(x) \le M$ on $[a,b]$, to give an estimate of the integral. (Use the graph of $x^2 - x$ to help you find $m$ and $M$.)

b) Evaluate the integral and verify your estimate from a).

a)




$-\frac{1}{4}(3-0) \le \int_0^3 x^2 - x\,dx \le 6(3-0)$

$-\frac{3}{4} \le \int_0^3 x^2 - x\,dx \le 18$

$x^2 - x = 0$ Max at $3 = 3^2 - 3 = 6$

$x(x-1) = 0$ min at $\frac{1}{2} = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$

$x = 0, 1$

$-\frac{1}{4} \le f(x) \le 6$

b) $\int_0^3 x^2 - x\,dx = \frac{x^3}{3} - \frac{x^2}{2}\Big|_0^3 = \frac{3^3}{3} - \frac{3^2}{2}$

$= 9 - \frac{9}{2} = \frac{9}{2}$, and $-\frac{3}{4} \le \frac{9}{2} \le 18$

**11.** (7pts) Write using sigma notation:

$\frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \cdots + \frac{14}{3^7} = \sum_{i=1}^{7} \frac{2i}{3^i}$

$2i$ $3^i$ $i = 1, \ldots 7$

**12.** (10pts) The rate at which a pool is filling is $\overset{4}{\cancel{6}}\sqrt[3]{t} \overset{-}{\cancel{+}} 2$ gallons per minute.
a) Use the Net Change Theorem to find the change in the amount of water in the pool from $t = 8$ to $t = 27$ minutes.
b) If at time $t = 8$ minutes there were 410 gallons of water in the pool, how much is in the pool at time $t = 27$ minutes?

a) $V'(t) = 4\sqrt[3]{t} - 2$

$$\int_8^{27} 4t^{\frac{1}{3}} - 2\,dt = \left(4\cdot\tfrac{3}{4}t^{\frac{4}{3}} - 2t\right)\Big|_8^{27} = \left(3t^{\frac{4}{3}} - 2t\right)\Big|_8^{27} \qquad 3^4 - 2^4$$

$$= 3\left(27^{\frac{4}{3}} - 8^{\frac{4}{3}}\right) - 2(27-8) = 3(81-16) - 38$$

$$= 3\cdot 65 - 38 = 195 - 38 = 157$$ gallons were added

b) At time 27 min the pool had $410 + 157 = 567$ gallons of water

**Bonus.** (10pts) A rocket takes off vertically from the ground, accelerating at constant acceleration. If at time $t = 5$ seconds it is at height 600 meters, what was its acceleration?

$a(t) = a$ $\qquad s(0) = 0,\ v(0) = 0$ $\qquad s(5) = 600$

$v(t) = at + C$

$0 = v(0) = a\cdot 0 + C$ so $C = 0$

$v(t) = at$

$s(t) = a\frac{t^2}{2} + D$

$0 = s(0) = a\cdot\frac{0^2}{2} + D$ so $D = 0$

$s(t) = \frac{a}{2}t^2$

$600 = s(5) = \frac{a}{2}\cdot 5^2$

$1200 = 25a$

$a = \frac{1200}{25} = \frac{12\cdot \overset{4}{\cancel{100}}}{\underset{1}{\cancel{25}}} = 48\ m/s^2$