Calculus 1 — Exam 2
MAT 250, Fall 2025 — D. Ivanšić

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Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\frac{d}{dx}\left(3x^7 - a^4 + \sqrt[5]{x^8} - \frac{7}{x^6}\right) = \frac{d}{dx}\left(3x^7 - a^4 + x^{\frac{8}{5}} - 7x^{-6}\right)$
$= 21x^6 + \frac{8}{5}x^{\frac{3}{5}} + 42x^{-7} = 21x^6 + \frac{8}{5}\sqrt[5]{x^3} + \frac{42}{x^7}$

2. (6pts) $\frac{d}{dx}\left(x\sqrt{x+3}\right) = 1 \cdot \sqrt{x+3} + x \cdot \frac{1}{2\sqrt{x+3}} = \frac{\sqrt{x+3} \cdot 2\sqrt{x+3} + x}{2\sqrt{x+3}}$
$= \frac{2(x+3)+x}{2\sqrt{x+3}} = \frac{3x+6}{2\sqrt{x+3}}$

3. (6pts) $\frac{d}{dt}\frac{t^2-4}{2t-3} = \frac{2t(2t-3)-(t^2-4) \cdot 2}{(2t-3)^2} = \frac{4t^2-6t-2t^2+8}{(2t-3)^2}$
$= \frac{2t^2-6t+8}{(2t-3)^2}$

4. (7pts) $\frac{d}{d\theta}\frac{\cos\theta}{\sin^3\theta} = \frac{-\sin\theta \sin^3\theta - \cos\theta \cdot 3\sin^2\theta\cos\theta}{(\sin^3\theta)^2} = \frac{-\sin^2\theta\left(\sin^2\theta + 3\cos^2\theta\right)}{\sin^6\theta}$
$= -\frac{\sin^2\theta + 3\cos^2\theta}{\sin^4\theta}$

5. (6pts) $\frac{d}{dx}\frac{1}{\cos(x^2-7)} = \frac{d}{dx}\left(\cos(x^2-7)\right)^{-1} = -\left(\cos(x^2-7)\right)^{-2}\left(-\sin(x^2-7)\right) \cdot 2x$
$= \frac{2x\sin(x^2-7)}{\cos^2(x^2-7)}$

6. (7pts) The limit at right represents a derivative $f'(a)$.
a) State f and a.
b) To find the limit, evaluate $f'(a)$ using differentiation rules.

$\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8} = \lim_{x\to 8}\frac{\sqrt[3]{x}-\sqrt[3]{8}}{x-8}$

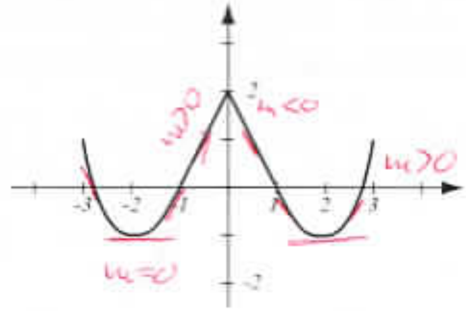
a) $f(x) = \sqrt[3]{x}$
$a = 8$

b) $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$
$f'(8) = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{8}^2} = \frac{1}{3} \cdot \frac{1}{2^2} = \frac{1}{12}$

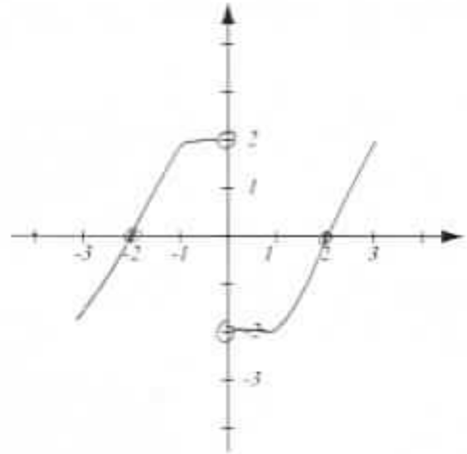
7. (10pts) The graph of the function $f(x)$ is shown at right.

a) Where is $f(x)$ not differentiable? Why?

b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.



a) at $x=0$, due to sharp point



8. (12pts) Let $f(x) = \dfrac{x}{x+1}$.

a) Use the limit definition of the derivative to find the derivative of the function.

b) Check your answer by taking the derivative of f using differentiation rules.

c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(-2, 2)$.

a) $\lim\limits_{x \to a} \dfrac{f(x)-f(a)}{x-a} = \lim\limits_{x\to a} \dfrac{\frac{x}{x+1} - \frac{a}{a+1}}{x-a} = \lim\limits_{x \to a} \dfrac{x(a+1)-a(x+1)}{(x+1)(a+1)} \cdot \dfrac{1}{x-a}$

$= \lim\limits_{x\to a} \dfrac{xa+x-ax-a}{(x-a)(x+1)(a+1)} = \lim\limits_{x\to a} \dfrac{\cancel{x-a}}{\cancel{(x-a)}(x+1)(a+1)} = \dfrac{1}{(a+1)(a+1)} = \dfrac{1}{(a+1)^2}$

b) $f'(x) = \dfrac{1\cdot(x+1) - x\cdot 1}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

c) $f'(-2) = \dfrac{1}{(-2+1)^2} = 1$

$y - 2 = 1\cdot(x-(-2))$

$y = x+2+2$

$y = x+4$

9. (10pts) Let $g(x) = \overset{\sin}{\cancel{\cos}}\, f(x)$ and $h(x) = \dfrac{x^3}{f(x)}$.

a) Find the general expressions for $g'(x)$ and $h'(x)$.

b) Use the table of values at right to find $g'(2)$ and $h'(4)$.

x	1	2	3	4
$f(x)$	-1	0	-3	2
$f'(x)$	1	-3	-1	0

a) $g'(x) = \cos(f(x)) \cdot f'(x)$ $\qquad g'(2) = \cos(f(2)) \cdot f'(2) = \cos 0 \cdot (-3) = -3$

b) $h'(x) = \dfrac{3x^2 f(x) - x^3 f'(x)}{f(x)^2}$ $\qquad h'(4) = \dfrac{3 \cdot 4^2 \cdot f(4) - 4^3 \cdot f'(4)}{f(4)^2}$

$$= \frac{3 \cdot 16 \cdot 2 - 4^3 \cdot 0}{2^2} = \frac{6 \cdot 16}{4} = 24$$

10. (6pts) An an arrow shot upwards has position (in meters, t in seconds) given by the formula $s(t) = -5t^2 + 60t$.

a) Write the formula for the velocity of the arrow at time t. What is the initial velocity of the arrow?

b) When does the arrow hit the ground? What is its velocity at that time?

a) $v(t) = s'(t) = -10t + 60$

$v(0) = -10 \cdot 0 + 60 = 60$ m/s

b) $-5t^2 + 60t = 0$ $\qquad v(12) = -10 \cdot 12 + 60 = -60$ m/s

$-5t(t-12) = 0$

$t = 12$

11. (10pts) Use implicit differentiation to find y' in general.

$x^2 + y^2 = \sqrt{xy}$ $\quad \Big| \dfrac{d}{dx}$

$2x + 2yy' = \dfrac{1}{2\sqrt{xy}}(1 \cdot y + xy')$ $\quad \Big| \cdot 2\sqrt{xy}$

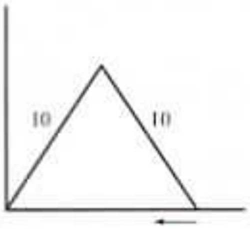
$4x\sqrt{xy} + 4y\sqrt{xy}\, y' = y + xy'$

$4x\sqrt{xy} - y = xy' - 4y\sqrt{xy}\, y'$

$4x\sqrt{xy} - y = y'(x - 4y\sqrt{xy})$

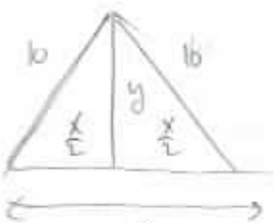
$y' = \dfrac{4x\sqrt{xy} - y}{x - 4y\sqrt{xy}}$

12. (14pts) A folding ladder whose sides are 10ft long has one end against a wall. If the other end is pushed toward the wall at rate 1/3 foot per second, how fast is the top of the ladder rising when the pushed end is 4 feet away from the wall?



Know: $x'(t) = -\frac{1}{3}$ ft/sec

Need: $y'(t)$ when $x = 4$



$$\left(\frac{x}{2}\right)^2 + y^2 = 10^2 \quad \Big| \frac{d}{dt}$$

$$\frac{\overset{1}{\cancel{2}}xx'}{\underset{2}{\cancel{4}}} + 2yy' = 0$$

$$2yy' = -\frac{xx'}{2}$$

$$y' = -\frac{xx'}{4y}$$

When $x = 4$:

$$\left(\frac{4}{2}\right)^2 + y^2 = 10^2$$

$$y^2 = 100 - 4 = 96$$

$$y = \pm\sqrt{96} = \pm 4\sqrt{6}, \quad y = 4\sqrt{6}$$

16·6

$$y' = -\frac{\cancel{4}\cdot\left(-\frac{1}{3}\right)}{\cancel{4}\cdot 4\sqrt{6}} = \frac{1}{12\sqrt{6}} \text{ ft/s}$$

Bonus. (10pts) Take the first five derivatives of $y = x\cos x$. Identify the pattern and use it to determine $y^{(59)}$.

$y = x\cos x$

$y' = 1\cos x + x(-\sin x) = \cos x - x\sin x$

$y'' = -\sin x - (1\cdot\sin x + x\cos x) = -2\sin x - x\cos x$

$y''' = -2\cos x - (1\cdot\cos x + x(-\sin x)) = -3\cos x + x\sin x$

$y^{(4)} = -3(-\sin x) + 1\cdot\sin x + x\cos x = 4\sin x + x\cos x$

$y^{(5)} = 4\cos x + 1\cdot\cos x + x(-\sin x) = 5\cos x - x\sin x$

$y^{(8)} = (4\sin x)^{(4)} + (x\cos x)^{(4)} = 4\sin x + 4\sin x + x\cos x$

$= 8\sin x + x\cos x$

Every 4th derivative has form

$n\sin x + x\cos x$ (n = how many derivatives)

$y^{(59)} = \left(y^{(56)}\right)^{(3)} = \left(56\sin x + x\cos x\right)^{(3)}$

$= -56\cos x + (-3\cos x + x\sin x)$

$= -59\cos x + x\sin x$