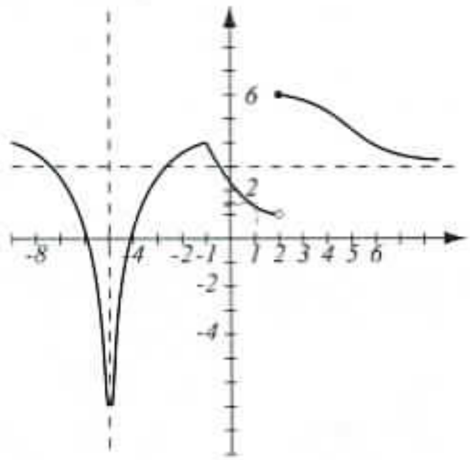
Calculus 1 — Exam 1
MAT 250, Fall 2025 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.



$\lim_{x\to-5} f(x) = -\infty$

$\lim_{x\to2-} f(x) = 1$

$\lim_{x\to2+} f(x) = 6$

$\lim_{x\to2} f(x) =$ DNE because one-sided limits are not same

$\lim_{x\to\infty} f(x) = 3$

$\lim_{x\to1} f(x) = 1.4$

List points in $(-\infty,\infty)$ where f is not continuous and justify why it is not continuous at those points.

Not cont at $x=-5$, $f(-5)$ not defined

$x=2$, $\lim_{x\to2} f(x)$ does not exist

2. (6pts) Let $\lim_{x\to4} f(x) = -2$ and $\lim_{x\to4} g(x) = 1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to4}\frac{2x+1-f(x)}{\sqrt{3+g(x)}} = \frac{\lim_{x\to4}(2x+1-f(x))}{\lim_{x\to4}\sqrt{3+g(x)}} = \frac{\lim_{x\to4}2x+\lim_{x\to4}1-\lim_{x\to4}f(x)}{\sqrt{\lim_{x\to4}(3+g(x))}} = \frac{2\lim_{x\to4}x+1-(-2)}{\sqrt{\lim_{x\to4}3+\lim_{x\to4}g(x)}}$$

$$= \frac{2\cdot4+3}{\sqrt{3+1}} = \frac{11}{2}$$

3. (10pts) Find $\lim_{x\to0} x^4\sqrt{\sin\frac{4}{x}+3}$. Use the theorem that rhymes with what students pay to a university in addition to tuition.

$-1 \le \sin\frac{4}{x} \le 1$

$2 \le \sin\frac{4}{x}+3 \le 4$

$\sqrt{2} \le \sqrt{\sin\frac{4}{x}+3} \le \sqrt{4}$

$\sqrt{2}x^4 \le x^4\sqrt{\sin\frac{4}{x}+3} \le 2x^4$

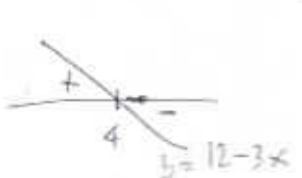
$\left.\begin{array}{l}\lim_{x\to0}\sqrt{2}x^4 = \sqrt{2}\cdot0^4 = 0\\ \lim_{x\to0}2x^4 = 2\cdot0^4 = 0\end{array}\right\}$ are equal, so by squeeze theorem

$\lim_{x\to0} x^4\sqrt{\sin\frac{4}{x}+3} = 0$

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\displaystyle \lim_{x\to\infty} \frac{x-7}{x^2+4x+13} = \lim_{x\to\infty} \frac{x\left(1-\overbrace{\tfrac{7}{x}}^{\to 0}\right)}{x^2\left(1+\underbrace{\tfrac{4}{x}}_{\to 0}+\underbrace{\tfrac{13}{x^2}}_{\to 0}\right)} = \lim_{x\to\infty} \frac{1}{x}\cdot\frac{1-0}{1+0+0} = 0\cdot 1 = 0$

5. (6pts) $\displaystyle \lim_{x\to 4^+} \frac{x-2}{12-3x} = \frac{2}{0^-} = -\infty$



OR $x > 4$

$3x > 12$

$0 > 12-3x$

OR $x = 4.01$

$12 - 3\cdot 4.01$

$= 12 - 12.03 = -0.03$

6. (5pts) $\displaystyle \lim_{x\to 7} \frac{2x^2-9x-35}{x-7} = \lim_{x\to 7} \frac{\cancel{(x-7)}(2x+5)}{\cancel{x-7}} = \lim_{x\to 7}(2x+5) = 2\cdot 7+5 = 19$

7. (7pts) $\displaystyle \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} = \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}}\cdot\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \lim_{x\to 2} \frac{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})}{\underbrace{\cancel{x-2}}_{\sqrt{x}^2-\sqrt{2}^2}}$

$\displaystyle = \lim_{x\to 2}(\sqrt{x}+\sqrt{2}) = 2\sqrt{2}$

8. (7pts) $\displaystyle \lim_{\theta\to 0} \frac{\tan(2\theta)}{3\theta} = \lim_{\theta\to 0} \frac{\frac{\sin(2\theta)}{\cos(2\theta)}}{3\theta} = \lim_{\theta\to 0} \frac{\sin(2\theta)}{\cos(2\theta)}\cdot\frac{1\cdot 2}{3\theta\cdot 2} = \lim$

$\displaystyle = \lim_{\theta\to 0} \frac{1}{\cos(2\theta)}\cdot\frac{\sin(2\theta)}{2\theta}\cdot\frac{2}{3} = \underbrace{\frac{1}{\cos 0}}_{=1}\cdot 1\cdot\frac{2}{3} = \frac{2}{3}$

9. (14pts) The equation $\sqrt{x+3} = x^2$ is given.

a) Use the Intermediate Value Theorem to show it has a solution in the interval $(2, 3)$.

b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

$\underbrace{\sqrt{x+3} - x^2}_{f(x),\text{ a continuous function}} = 0$

a) $f(2) = \sqrt{2+3} - 4 = \sqrt{5} - 1 > 0$

$f(3) = \sqrt{3+3} - 9 = \sqrt{6} - 6 < 0$

Since $f(3) < 0 < f(2)$

by IVT there is a c in $(2,3)$

such that $f(c) = 0$

b) $f(2.11) = 0.0004539$

$f(2.12) = -0.018378$

Since $f(2.12) < 0 < f(2.11)$

by IVT there is a c in $(2.11, 2.12)$

such that $f(c) = 0$.

10. (10pts) Consider the limit $\lim_{x \to 3} \dfrac{2^x - 8}{x - 3}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values (no more than 5 per table) that will support your answer.

x	$\frac{2^x - 8}{x - 3}$
3.1	5.741877
3.01	5.564440
3.001	5.5471
3.0001	5.54537
3.00001	5.5452

x	$\frac{2^x - 8}{x - 3}$
2.9	5.357736
2.99	5.526004
2.999	5.543256
2.9999	5.544985
2.99999	5.545158

It appears that

$\lim_{x \to 3} \dfrac{2^x - 8}{x - 3} \approx 5.545$

11. (12pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(-\infty, 3)$ and $(3, \infty)$.
b) For which c is the function continuous at point $x = 3$?

$$f(x) = \begin{cases} c - x^2, & \text{if } x \le 3 \\ -\frac{6x}{c}, & \text{if } x > 3. \end{cases}$$

a) On $(-\infty,3)$ f is a polynomial, On $(3,\infty)$ f is a polynomial $\Big\}$ hence, continuous

Must have $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(c-x^2) = c-9$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} -\frac{6x}{c} = -\frac{18}{c}$, $f(3) = c-9$

Must have

$c-9 = -\frac{18}{c}$ i.e. $c^2-9c+18=0$

$c^2-9c=-18$ $\quad (c-3)(c-6)=0$

$c=3,6$ makes the function continuous

Bonus. (10pts) Find the limit.

$$\lim_{x\to\infty}\left(\sqrt{2x^2+5x+1}-\sqrt{2x^2-3x+2}\right) = \lim_{x\to\infty} \sqrt{2x^2+5x+1}-\sqrt{2x^2-3x+2}\cdot\frac{\sqrt{2x^2+5x+1}+\sqrt{2x^2-3x+2}}{\sqrt{2x^2+5x+1}+\sqrt{2x^2-3x+2}}$$

$$= \lim_{x\to\infty}\frac{2x^2+5x+1-(2x^2-3x+2)}{\sqrt{x^2(2+\frac{5}{x}+\frac{1}{x^2})}+\sqrt{x^2(2-\frac{3}{x}+\frac{2}{x^2})}} = \lim_{x\to\infty}\frac{8x-1}{x\sqrt{2+\frac{5}{x}+\frac{1}{x^2}}+x\sqrt{2-\frac{3}{x}+\frac{2}{x^2}}}$$

may assume $x>0$ since $x\to\infty$, so $\sqrt{x^2}=x$

$$= \lim_{x\to\infty}\frac{x(8-\underbrace{(\tfrac{1}{x})}_{\to 0})}{x\left(\sqrt{2+\underbrace{(\tfrac{5}{x})}_{\to 0}+\underbrace{(\tfrac{1}{x^2})}_{\to 0}}+\sqrt{2-\underbrace{(\tfrac{3}{x})}_{\to 0}+\underbrace{(\tfrac{2}{x^2})}_{\to 0}}\right)} = \frac{8-0}{\sqrt{2+0+0}+\sqrt{2-0+0}} = \frac{8}{2\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$