Trigonometry — Joysheet 1
MAT 145, Fall 2025 — D. Ivanšić

Name: _____

Covers: 6.1, 6.2 *Show all your work!*

1. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\cos\theta = \frac{2}{5}$.

2. (8pts) If θ is an acute angle, find the values of all the trigonometric functions of θ given that $\tan\theta = w$, where w is some number.

3. (10pts) Given that $\cos 17^\circ = a$, $\tan 35^\circ = b$, $\cot 11^\circ = c$ and $\csc 53^\circ = d$, use basic and cofunction identities to express the following quantities using a, b, c and d.

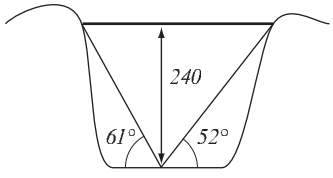
$\cot 35^\circ =$ $\sin 73^\circ =$ $\tan 79^\circ =$ $\sec 17^\circ =$

$\cot 55^\circ =$ $\sin 53^\circ =$ $\sec 37^\circ =$ $\tan 11^\circ =$

4. (10pts) Solve the right triangle (that is, find all sides and angles), if $a = 3$, $c = 8$.

5. (10pts) From the rim of a canyon, you observe a painted rock at the bottom of the canyon at angle of depression 22°. If you know the canyon is 315 meters deep, what is the distance from you to the painted rock?

6. (14pts) Standing at a point 240 feet under a bridge, the ends of the bridge appear at angles of elevation 52° and 61°. How long is the bridge?



Trigonometry — Joysheet 2
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Name: _____

Covers: 6.3, 6.4 *Show all your work!*

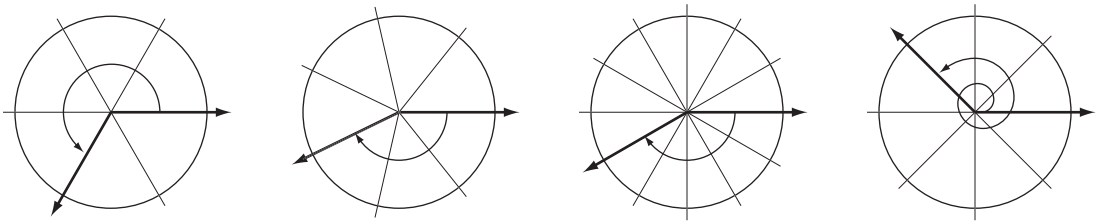
1. (9pts) If $\sec\theta = \dfrac{5}{\sqrt{2}}$ and θ is in the fourth quadrant, find the exact values of all the trigonometric functions of θ. Draw a picture.

2. (7pts) The terminal side of angle θ is in the second quadrant and lies on the line $2x + 7y = 0$. Find the exact values of $\sin\theta$ and $\tan\theta$. Draw a picture.

3. (8pts) Sketch angles in standard position with indicated radian measure.

$$\frac{7\pi}{6} \qquad -\frac{4\pi}{7} \qquad \frac{9\pi}{4} \qquad -\frac{7\pi}{9}$$

4. (8pts) Indicate both the radian and degree measure under the following angles. (Use equally-spaced lines to help you determine what the angles are.)



5. (8pts) What is the length of the arc that the tip of the minute hand traces out while it travels an hour and 10 minutes? The length of the minute hand is 6 cm.

6. (8pts) What is the distance from Chicago, IL (latitude $41°52'17''$) to Tegucigalpa, Honduras (latitude $14°4'15''$) which lies directly south of Chicago? Radius of Earth is 3960 miles.

7. (12pts) The diameter of a car tire is 32 inches. Suppose the car is traveling along the highway at 70 mph (thus, you may assume that every point at the edge of the tire has this linear speed).
a) What is the angular speed of the wheel in radians per minute?
b) How many revolutions per minute does the wheel make?

Trigonometry — Joysheet 3
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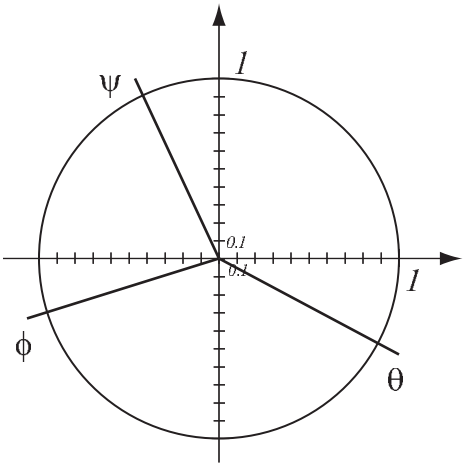
Name: _____

Covers: 6.5, 6.6 *Show all your work!*

1. (12pts) Use the unit circle to estimate the values of the trigonometric functions of the angles drawn.



$\cos\psi =$ $\csc\psi =$

$\sin\phi =$ $\tan\phi =$

$\sin\theta =$ $\sec\theta =$

2. (18pts) For each of the following, draw the unit circle and the appropriate angle in order to infer from the picture the exact values of the trigonometric functions.

$\sin 150° =$ $\cos\dfrac{4\pi}{3} =$ $\tan 90° =$

$\sec(-225°) =$ $\csc\dfrac{11\pi}{6} =$ $\cot\dfrac{3\pi}{2} =$

$\cos(-570°) =$ $\tan\left(-\dfrac{5\pi}{4}\right) =$ $\sec\dfrac{41\pi}{3} =$

3. (8pts) Draw two periods of the graph of $y = 2\sin(4x)$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

4. (11pts) Draw two periods of the graph of $y = -\cos\left(2x + \frac{2\pi}{3}\right)$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

5. (11pts) Draw two periods of the graph of $y = 4\sin(-3x + \pi) - 2$.
What is the amplitude? The period?
For each period, indicate x-coordinates of the five special points (middle, peaks, valleys).

Trigonometry — Joysheet 4
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Name: _____

Covers: 7.1, 7.2, 7.3 Show all your work!

1. (10pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$ are angles so that $\cos\alpha = -\frac{1}{3}$ and $\cos\beta = \frac{2}{5}$. Find the exact value of $\cos(\alpha + \beta)$.

2. (6pts) Show the identity in two ways:
1) algebraically 2) with a picture in which $\frac{\pi}{2} < \theta < \pi$

$\sin(\pi - \theta) = \sin\theta$

3. (8pts) Use a half-angle formula to find the exact value of $\sin\dfrac{9\pi}{8}$ (do not use the calculator).

4. (8pts) Use identities to simplify the following expressions.

$2\sin\theta\,\sin\left(\dfrac{\pi}{2} - \theta\right) =$

$$\frac{\cos^2\theta\,\sin^2\left(\frac{\pi}{2} - \theta\right) - \sin^2\theta\,\cos^2\left(\frac{\pi}{2} - \theta\right)}{\cos^2\theta - \cos^2\left(\frac{\pi}{2} - \theta\right)} =$$

5. (8pts) Show the identity.

$\tan\theta - \cot\theta = (\sec\theta - \csc\theta)(\sin\theta + \cos\theta)$

6. (10pts) Show the identity.

$\dfrac{1+\sin\theta}{1-\sin\theta} = (\sec\theta + \tan\theta)^2$

7. (10pts) Develop the formula for $\sin(5\theta)$ by starting as follows and using sum and double-angle identities. The final expression should only have $\sin\theta$ and $\cos\theta$ in it.

$\sin(5\theta) = \sin(3\theta + 2\theta) =$

Trigonometry — Joysheet 5
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Name: _____

Covers: 7.4, 7.5 *Show all your work!*

1. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\arcsin\left(-\frac{1}{2}\right) =$ $\arctan\sqrt{3} =$ $\arccos(1.2) =$

2. (7pts) Find the exact value of the expressions (do not use the calculator). For some of them, you will need a picture.

$\cos(\arccos 0.31) =$ $\arccos\left(\cos\left(-\frac{4\pi}{5}\right)\right) =$ $\arcsin\left(\sin\frac{5\pi}{7}\right) =$

3. (5pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$\tan\left(\arccos\left(-\frac{3}{8}\right)\right) =$

4. (5pts) Solve the equation (give a general formula for all solutions).

$2\sin\theta - \sqrt{3} = 0$

5. (5pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\sin\theta = -0.7$

6. (10pts) Solve the equation and give a general formula for all solutions. Then list all the solutions that fall in the interval $[0, 2\pi)$.

$2\cos^2\theta + \cos\theta - 1 = 0$

7. (6pts) Solve the equation on the interval $[0, 2\pi)$.

$2\sin^2\theta - \cos(2\theta) = 2$

8. (7pts) Solve the equation (give a general formula for all the solutions).

$2\sec^2\theta + 4\sec\theta = 3 - \tan^2\theta$

9. (7pts) Find the exact value of the expression (do not use the calculator).

$\tan\left(\frac{\pi}{4} + \arccos\left(-\frac{2}{5}\right)\right) =$

Trigonometry — Joysheet 6
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Name: _____

Covers: 8.1, 8.2 *Show all your work!*

1. (7pts) Solve the triangle: $c = 6$, $A = 34°$, $B = 47°$.

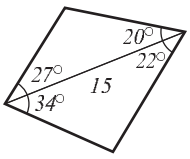
2. (12pts) Solve the triangle: $b = 11$, $c = 9$, $C = 31°$

3. (7pts) Solve the triangle: $b = 3$, $c = 7$, $A = 101°$.

4. (12pts) A 120 ft tall light house stands on top of a hill. A ship observes the angles of elevation to the bottom and top of the lighthouse as 19° and 28°. How tall is the hill? (Draw a picture.)

5. (10pts) Port B is located 34 kilometers N25°E from port A. A ship is 97 kilometers from port B and has position N66°E of port B. How far is the ship from port A (draw a picture)?

6. (12pts) Determine the area of the pictured quadrangle: one of its diagonals has length 15 units.



Trigonometry — Joysheet 7
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Name: _____

Covers: 8.4 *Show all your work!*

1. (12pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(3, -\frac{5\pi}{6}\right)$ $(r, \theta) = \left(-4, \frac{3\pi}{4}\right)$ $(r, \theta) = \left(6, \frac{5\pi}{2}\right)$

2. (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one with a positive r and at least one with a negative r. Give exact answers — do not use the calculator.

$(x, y) = (-5\sqrt{3}, 15)$ $(x, y) = (-3, -3)$ $(x, y) = (-4, -7)$

3. (6pts) Convert to a polar equation.

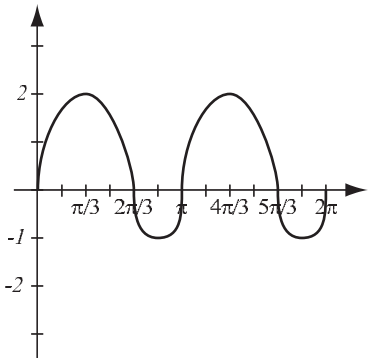
$x^2 - 2x + y^2 - 2y + 1 = 0$

4. (6pts) Convert to a rectangular equation.

$r = \cos(2\theta) + \sin(2\theta)$

5. (12pts) Graph the equation $r = 2\sin\theta - 1$ by doing the following:
a) Graph the equation in rectangular r-θ coordinates.
b) Use the information from a) to help you graph the equation in polar coordinates. Indicate corresponding parts of the graph in a) and b). Check your work with the graphing calculator.

6. (8pts) Below is the graph of the function $r = f(\theta)$ in rectangular $r - \theta$ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates indicating corresponding parts of the graphs.



7. (4pts) Use your calculator or a graphing program (look online) to draw an accurate graph of the polar curve.

$r = 5 + 3\cos(8\theta)$