Trigonometry — Exam 3
MAT 145, Spring 2025— D. Ivanšić

Name: _____

Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2 \sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$

$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ $\quad \tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (6pts) Solve the triangle: $b = 5$, $c = 3$, $C = 40°$.

2. (12pts) Solve the triangle: $a = 4$, $c = 7$, $B = 112°$

3. (14pts) Solve the triangle: $a = 7$, $c = 2$, $A = 67°$.

4. (8pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(4, \frac{3\pi}{4}\right)$ $(r, \theta) = \left(-2, -\frac{7\pi}{6}\right)$

5. (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give three answers in polar coordinates, at least one of which has a negative r. Give exact answers — do not use the calculator.

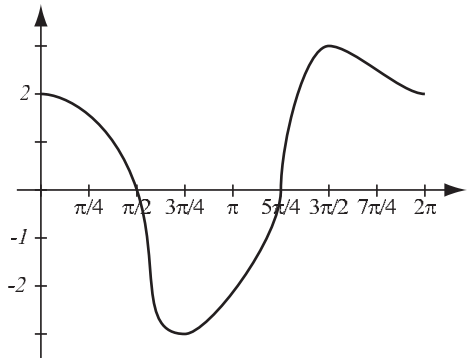
$(x, y) = (-5, -5)$ $(x, y) = (3\sqrt{3}, -3)$

6. (8pts) Convert to a polar equation. Answer should be solved for r.

$x^2 - 2x + y^2 = 1$

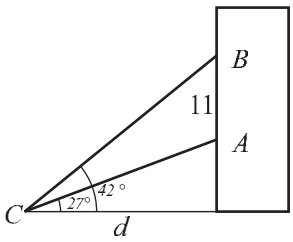
7. (8pts) Determine the distance between points given in **polar coordinates**: $B = \left(4, \frac{2\pi}{3}\right)$, $C = \left(2, \frac{\pi}{3}\right)$.
a) Draw the picture.
b) Find the exact distance from B to C (do not use the calculator).

8. (8pts) Below is the graph of the function $r = f(\theta)$ in rectangular r-θ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates, indicating corresponding parts of the graphs.



9. (10pts) If the long hand of the clock has length 5 in and the short hand 3 in, what is the distance between the tips of the hands at 7 o'clock?

10. (14pts) To determine distance d to a building, sightings of points A and B on the building are made and they stand at angles of elevation $27°$ and $42°$. It is known that the distance from A to B is 11 meters.
a) Determine angles in the triangle ABC.
b) Find the distance to the building d.



Bonus. (10pts) Show that the area of a rectangle is one-half of product of lengths of diagonals times the sine of the angle between them.