**Trigonometry — Exam 2**
**MAT 145, Spring 2025— D. Ivanšić**

**Name:** _____

*Show all your work!*

| | |
|---|---|
| $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$ | $\sin(2u) = 2 \sin u \cos u$ |
| $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$ | $\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ |
| $\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$ | $\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$ |
| $\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$ | $\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$ |

**1.** (16pts) Use an identity (sum, difference, half- or double-angle) to find the exact values of the trigonometric functions below (do not use the calculator).

$\sin \dfrac{11\pi}{12} =$

$\cos 67.5^\circ =$

**2.** (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arccos(-3) =$ $\qquad$ $\arccos \dfrac{\sqrt{3}}{2} =$ $\qquad$ $\arcsin\left(-\dfrac{1}{2}\right) =$ $\qquad$ $\arctan(-1) =$

**3.** (6pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

$\tan(\arctan 3.7) =$ $\qquad$ $\arcsin\left(\sin \dfrac{7\pi}{5}\right) =$

**4.** (6pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\tan\left(\arccos\left(-\frac{3}{4}\right)\right) =$$

**5.** (8pts) Use identities to simplify the following expression.

$$\left(\sin\left(\frac{\pi}{2}-\theta\right)+\sin\theta\right)\left(\cos\theta-\cos\left(\frac{\pi}{2}-\theta\right)\right) =$$

Show the identities:

**6.** (8pts) $\sin\theta(\csc\theta-\sin\theta)=\cos^2\theta$

**7.** (10pts) $\tan\theta+\cot\theta=\dfrac{2}{\sin(2\theta)}$

**8.** (5pts) Solve the equation in radians (state general solution).

$4\sin\theta - 2\sqrt{2} = 0$

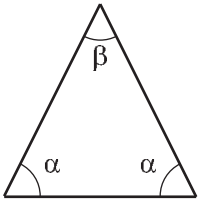
**9.** (7pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\sin\theta = -0.33$

**10.** (14pts) Solve the equation in radians.
a) State the general solution.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$6\cos^2\theta - \cos\theta - 1 = 0$

**11.** (12pts) An isosceles triangle has two sides of equal length and two angles of same measure. If an isosceles triangle has angles $\alpha$, $\alpha$ and $\beta$, and $\cos\alpha = \frac{1}{3}$, find the exact value of $\cos\beta$ (do not use the calculator).




**Bonus.** (10pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{3\pi}{2} < \beta < 2\pi$ are angles so that $\cos\alpha = -\frac{2}{5}$ and $\cos\beta = \frac{2}{7}$. Find the exact value of $\sin(2\alpha + 2\beta)$.