Trigonometry — Exam 3
MAT 145, Fall 2025— D. Ivanšić

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Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2\sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\quad \sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (6pts) Solve the triangle: $a = 9$, $b = 5$, $c = 3$.

$3+5<9$

so no solution

or: $\cos C = \frac{9^2+5^2-3^2}{2 \cdot 9 \cdot 5} = \frac{81+25-9}{90} = \frac{95}{90} > 1$

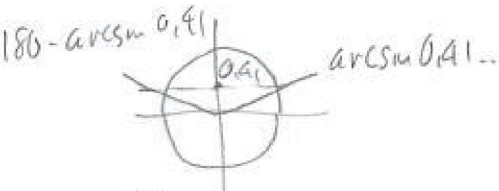
so there is no solution

2. (14pts) Solve the triangle: $b = 5$, $c = 3$, $B = 44°$.

$\frac{5}{\sin 44^\circ} = \frac{3}{\sin C}$

$5\sin C = 3\sin 44^\circ$

$\sin C = \frac{3\sin 44^\circ}{5} = 0.416795$



① $C = \arcsin 0.41.. = 24.632409^\circ$

$A = 180^\circ - 44^\circ - 24.632^\circ = 111.367591$

$\frac{a}{\sin 111.36.} = \frac{5}{\sin 44^\circ}$

$a = \frac{5\sin 111.36}{\sin 44^\circ} = 6.703022$

② $C = 180 - 24.632.. = 155.36759$

Too big, as $B + C > 180^\circ$ no solution

$44 + 155$

3. (14pts) Solve the triangle: $b = 5$, $c = 2$, $A = 32°$

$a^2 = 5^2 + 2^2 - 2 \cdot 5 \cdot 2 \cos 32^\circ$

$a^2 = 12.03..$, $a = \sqrt{12.03} = 3.469732$

$\cos B = \frac{3.46..^2 + 2^2 - 5^2}{2 \cdot 3.46.. \cdot 2} = -0.6456..$

$B = \arccos(-0.6456) = 130.214601$

$A = 180^\circ - 32^\circ - 130.214603$

$= 17.785395$

4. (~~5~~ 10pts) Draw points with the following polar coordinates. Then convert them into rectangular coordinates. Give exact answers — do not use the calculator.

$(r, \theta) = \left(3, \frac{4\pi}{3}\right)$

$x = 3 \cdot \cos\frac{4\pi}{3} = 3 \cdot \left(-\frac{1}{2}\right) = -\frac{3}{2}$

$y = 3 \cdot \sin\frac{4\pi}{3} = 3\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$

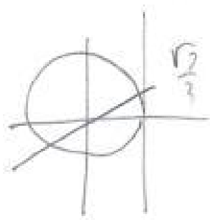
$(r, \theta) = \left(-5, \frac{5\pi}{4}\right)$

$x = -5\cos\frac{5\pi}{4} = -5\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$

$y = -5\sin\frac{5\pi}{4} = -5\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2}$

5. (12pts) Convert the following rectangular coordinates into polar coordinates. Draw a picture to make sure you have the correct θ. For each point, give two answers in polar coordinates, one with a positive r and one with a negative r. Give exact answers — do not use the calculator.

$(x, y) = (-6, -2\sqrt{3})$



$r = \sqrt{(-6)^2 + (-2\sqrt{3})^2}$

$= \sqrt{36 + 12}$

$= \sqrt{48} = 4\sqrt{3}$

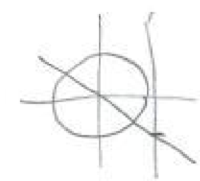
$\tan\theta = \frac{-2\sqrt{3}}{-6} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ $\quad \theta = \frac{\pi}{6}$ or $\frac{2\pi}{6}$

16.3

$\left(4\sqrt{3}, \frac{2\pi}{6}\right)$

or $\left(-4\sqrt{3}, \frac{\pi}{6}\right)$

$(x, y) = (-2, 2)$



$r = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{2}{-2} = -1$

$\theta = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\left(2\sqrt{2}, \frac{3\pi}{4}\right)$

$\left(-2\sqrt{2}, -\frac{\pi}{4}\right)$

6. (~~5~~ 10pts) Convert to a rectangular equation.

$r = \sec\theta + \csc\theta \quad | \cdot r$

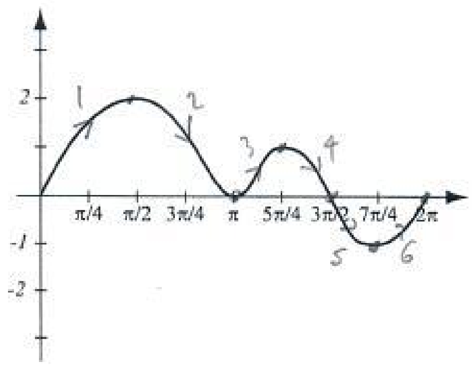
$r = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \quad | \cdot \sin\theta\cos\theta$

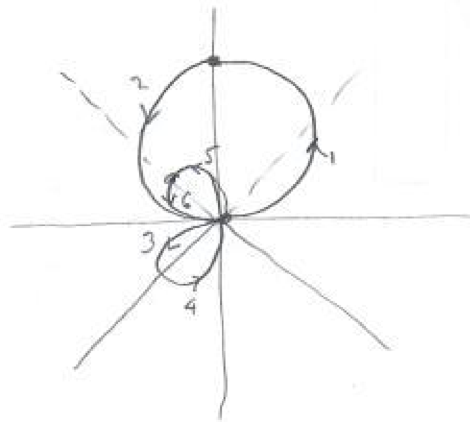
$r\sin\theta\cos\theta = \sin\theta + \cos\theta \quad | \cdot r$

$r\sin\theta \, r\cos\theta = r\sin\theta + r\cos\theta$

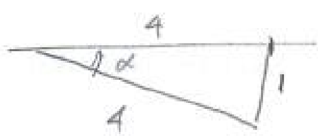
$xy = x + y$

7. (10pts) Below is the graph of the function $r = f(\theta)$ in rectangular r-θ coordinates. Use the graph to draw the graph of $r = f(\theta)$ in polar coordinates, indicating corresponding parts of the graphs.





8. (10pts) You opened a 4-foot wide door so that the vertical edge of the door is 1 foot away from the frame. What is the angle of the door to its closed position? (Draw a picture viewing the door from above.)



Law of cosines

$\alpha = ?$

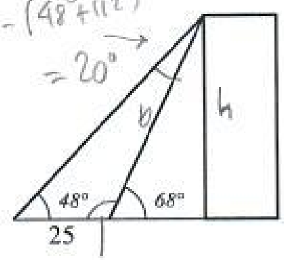
$$\cos\alpha = \frac{4^2+4^2-1^2}{2\cdot 4\cdot 4} = \frac{31}{32}$$

$$\alpha = \arccos\frac{31}{32} = 14.361512^\circ$$

9. (14pts) From one point, you find the angle of elevation to the top of the building to be 48°. After moving 25 meters toward the building, you find the angle of elevation to the top of the building is 68°. How tall is the building?

$180° - (48° + 112°)$

$= 20°$



$180° - 68°$

$= 112°$

$\frac{h}{b} = \sin 68°$

$h = b \sin 68° = 54.32.. \cdot \sin 68° = 50.364861$ meter

$\frac{b}{\sin 48°} = \frac{25}{\sin 20°}$, so $b = \frac{25 \sin 48°}{\sin 20°} = 54.320253$ meters

Bonus. (10pts) Convert the formula for distance between points (x_1, y_1) and (x_2, y_2) in Cartesian coordinates:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

into polar coordinates (r_1, θ_1) and (r_2, θ_2) of those same points to get the formula for distance between points given by polar coordinates:

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)}$$

$$d = \sqrt{(r_2 \cos\theta_2 - r_1 \cos\theta_1)^2 + (r_2 \sin\theta_2 - r_1 \sin\theta_1)^2}$$

$$= \sqrt{r_2^2 \cos^2\theta_2 - 2 r_1 r_2 \cos\theta_2 \cos\theta_1 + r_1^2 \cos^2\theta_1 + r_2^2 \sin^2\theta_2 - 2 r_1 r_2 \sin\theta_2 \sin\theta_1 + r_1^2 \sin^2\theta_1}$$

$$= \sqrt{r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_1^2(\cos^2\theta_1 + \sin^2\theta_1) - 2 r_1 r_2 (\cos\theta_2 \cos\theta_1 + \sin\theta_2 \sin\theta_1)}$$

$$= \sqrt{r_2^2 + r_1^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)}$$