Trigonometry — Exam 2
MAT 145, Fall 2025— D. Ivanšić

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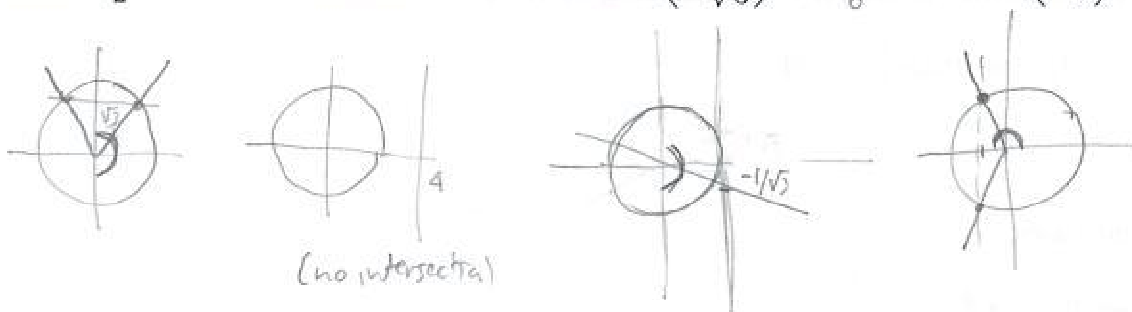
Show all your work!

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	$\sin(2u) = 2\sin u \cos u$
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$
$\cos^2 \frac{u}{2} = \frac{1+\cos u}{2}$ $\sin^2 \frac{u}{2} = \frac{1-\cos u}{2}$	$\tan^2 \frac{u}{2} = \frac{1-\cos u}{1+\cos u}$

1. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

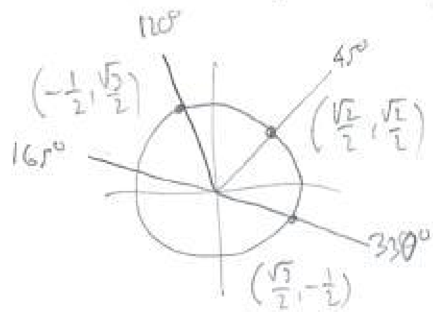
$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ $\arccos 4 =$ not def. $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



2. (10pts) Use an identity (sum, difference, half- or double-angle) to find the exact value of the trigonometric function (do not use the calculator).

$\cos 165° = \cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}-\sqrt{6}}{4} = -\frac{\sqrt{2}+\sqrt{6}}{4}$



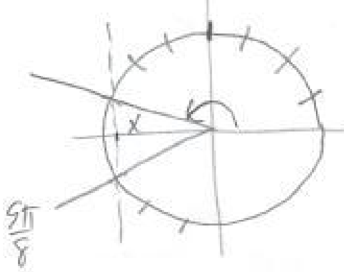
OR: $\cos^2 \frac{330°}{2} = \frac{1+\cos 330°}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} \cdot \frac{2}{2} = \frac{2+\sqrt{3}}{4}$

$\cos 165° = \pm\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$

due to Q II

3. (7pts) Find the exact value of the expressions (do not use the calculator). For one of them, you will need a picture.

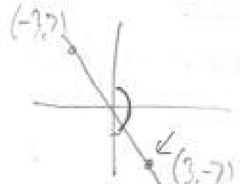
$\sin(\arcsin 0.27) = 0.27$ $\arccos\left(\underbrace{\cos \frac{9\pi}{8}}_{x}\right) = \arccos x = \frac{7\pi}{8}$



4. (8pts) Find the exact value of the expression (do not use the calculator). Draw the appropriate picture.

$$\sin\Big(\underbrace{\arctan\left(-\frac{7}{3}\right)}_{=\theta}\Big) = \quad \sin\theta = \frac{-7}{\sqrt{58}} = -\frac{7}{\sqrt{58}}$$

so $\tan\theta = -\frac{7}{3} = \frac{-7}{3} = \frac{7}{-3}$



$r = \sqrt{3^2 + (-7)^2} = \sqrt{9+49} = \sqrt{58}$

5. (8pts) Use identities to simplify the following expression.

$$\cos\theta\cos\left(\frac{\pi}{2}-\theta\right) + \sin\theta\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta\sin\theta + \sin\theta\cos\theta = 2\sin\theta\cos\theta = \sin(2\theta)$$

Show the identities:

6. (8pts) $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin\theta\cdot\sin\theta}{\cos\theta\cdot\sin\theta} + \frac{\cos\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta} = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

7. (10pts) $(\tan\theta + \sec\theta)(\cot\theta + \csc\theta) = (\sec\theta + 1)(\csc\theta + 1)$

$$(\tan\theta + \sec\theta)(\cot\theta + \csc\theta) = \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right) =$$

$$= \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta} + \frac{\cancel{\cos\theta}^{\,1}}{\cancel{\cos\theta}\sin\theta} + \frac{\cancel{\sin\theta}^{\,1}}{\cos\theta\cancel{\sin\theta}} + \frac{1}{\cos\theta\sin\theta}$$

$$= 1 + \frac{1}{\sin\theta} + \frac{1}{\cos\theta} + \frac{1}{\cos\theta\sin\theta}$$

$$= 1 + \csc\theta + \sec\theta + \sec\theta\csc\theta = (1+\sec\theta)(1+\csc\theta)$$

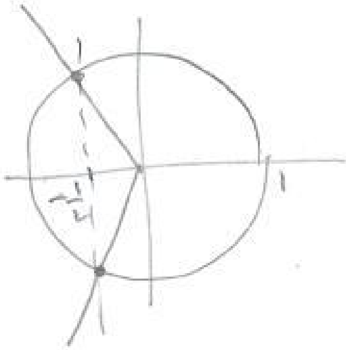
8. (6pts) Solve the equation in radians (state general solution).

$6\cos\theta + 3 = 0$

$6\cos\theta = -3$

$\cos\theta = -\frac{3}{6}$

$\cos\theta = -\frac{1}{2}$

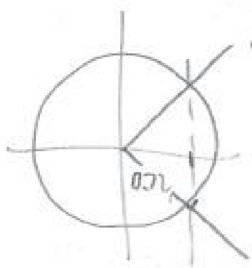


$\theta = \frac{2\pi}{3} + k\cdot 2\pi$

$\theta = -\frac{2\pi}{3} + k\cdot 2\pi$

9. (8pts) Use your calculator to solve the equation on the interval $[0°, 360°)$ (answers in degrees). A picture will help.

$\cos\theta = 0.72$



$\arccos(0.72) = 43.94552$

$360° - \arccos 0.72 = 316.054480$

10. (14pts) Solve the equation in radians.
a) State the general solution.
b) List all the solutions that fall in the interval $[0, 2\pi)$.

$2\sin^2\theta + 7\sin\theta - 4 = 0$

Let $u = \sin\theta$

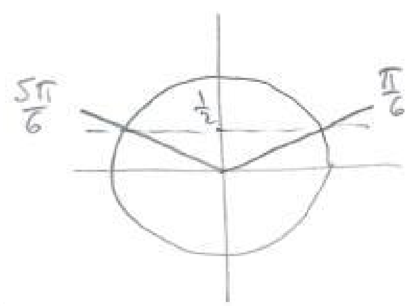
$2u^2 + 7u - 4 = 0$

$49+32$

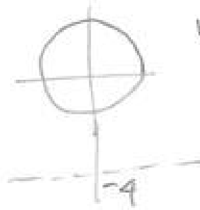
$u = \frac{-7 \pm \sqrt{7^2 - 4\cdot 2\cdot(-4)}}{2\cdot 2}$

$= \frac{-7 \pm \sqrt{81}}{4} = \frac{-7 \pm 9}{4} = -4, \frac{1}{2}$

$\sin\theta = \frac{1}{2}$



$\sin\theta = -4$

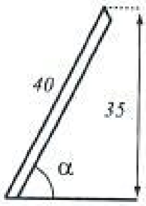


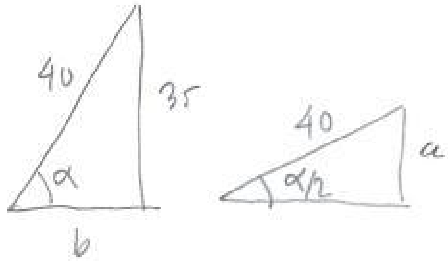
no solution

a) $\theta = \frac{\pi}{6} + k\cdot 2\pi, \ \theta = \frac{5\pi}{6} + k\cdot 2\pi$

b) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

11. (13pts) When a 40-foot long crane is raised to angle α with the horizontal, its top is 35 feet from the ground. How high above ground is the top of the crane, if it is lowered to angle $\frac{\alpha}{2}$ with the horizontal? Give exact value and do not use the calculator.





$\sin\alpha = \frac{35}{40} = \frac{7}{8}$ $\qquad \sin\frac{\alpha}{2} = \frac{a}{40}$, so $a = 40\sin\frac{\alpha}{2} = 40\,\frac{\sqrt{8-\sqrt{15}}}{4} = 10\sqrt{8-\sqrt{15}}$

$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1-\frac{\sqrt{15}}{8}}{2}\cdot\frac{8}{8} = \frac{8-\sqrt{15}}{16}$ $\qquad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{8-\sqrt{15}}{16}} = \frac{\sqrt{8-\sqrt{15}}}{4}$

since $\frac{\alpha}{2} < 90^\circ$

$b^2 + 35^2 = 40^2$

$b^2 = 40^2 - 35^2 = 1600 - 1225 = 375$

$b = \pm\sqrt{375} = \pm 5\sqrt{15}$

$25\cdot 15$

$b = 5\sqrt{15}$ since $b > 0$

$\cos\alpha = \frac{5\sqrt{15}}{40} = \frac{\sqrt{15}}{8}$

Bonus. (10pts) Develop the formula for $\sin(3\theta)$ by starting as follows and using sum and double-angle identities. Transform it so that the final expression only has $\sin\theta$ in it.

$\sin(3\theta) = \sin(2\theta + \theta) = \sin(2\theta)\cos\theta + \cos(2\theta)\sin\theta$

use this version since it needs to be in terms of $\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$