1. (8pts) Use the graph of the function f at right to answer the following questions.
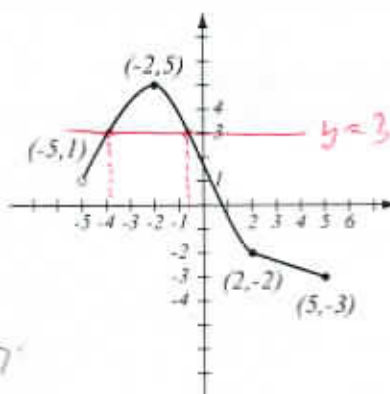
a) Find: $f(2) = -2$ $f(-5) =$ not def.

b) What is the domain of f? $(-5, 5]$

c) What is the range of f? $[-3, 5]$

d) What are the solutions of the equation $f(x) = 3$? $x = -3.8, -0.7$

2. (12pts) Use your calculator to accurately sketch the graph of $f(x) = x^3 - 3x^2 + 5$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) Determine the range of the function in interval notation by looking at the graph.

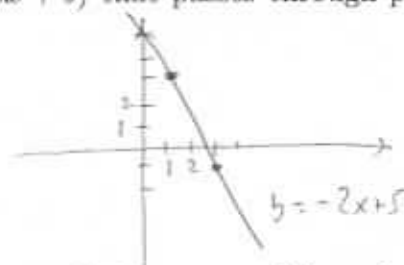y-int: $f(0) = 5$, x-int: -1.103803
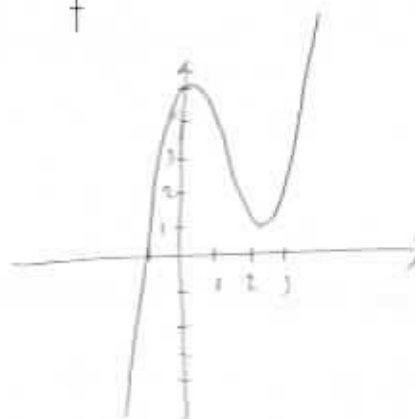
Range $= (-\infty, \infty)$

3. (5pts) Find the equation of the line (in form $y = mx + b$) that passes through points $(1, 3)$ and $(3, -1)$. Draw the requested line.

$m = \dfrac{-1-3}{3-1} = \dfrac{-4}{2} = -2$

$y - 3 = -2(x-1)$

$y = -2x + 2 + 3$

$y = -2x + 5$

$y = -2x + 5$

4. (9pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $3x + 4y = 20$ and contains the point $(4, -1)$. Draw both lines.

$3x + 4y = 20$
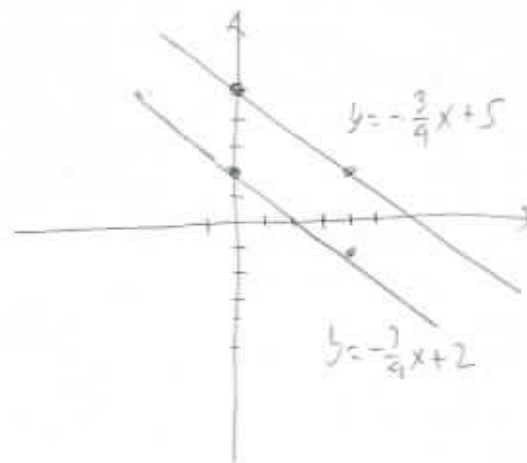
$4y = -3x + 20 \mid \div 4$

$y = -\frac{3}{4}x + 5$

$m = -\frac{3}{4}$

Same slope for parallel line

$y - (-1) = -\frac{3}{4}(x-4)$

$y + 1 = -\frac{3}{4}x + 3$

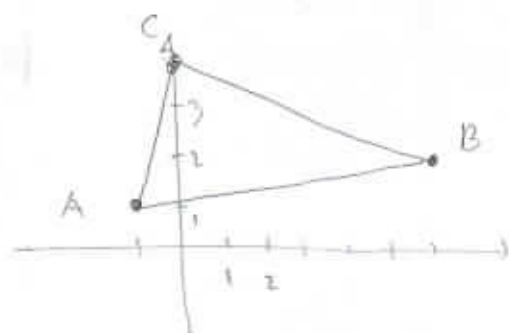$y = -\frac{3}{4}x + 2$

$y = -\frac{3}{4}x + 5$

$y = -\frac{3}{4}x + 2$

5. (9pts) In a coordinate system, draw the triangle with vertices $A = (-1, 1)$, $B = (6, 2)$ and $C = (0, 4)$.

a) Compute the slopes of all sides.

b) Use slopes to determine if the triangle is a right triangle.

a) $m_{AB} = \dfrac{2-1}{6-(-1)} = \dfrac{1}{7}$

$m_{BC} = \dfrac{4-2}{0-6} = \dfrac{2}{-6} = -\dfrac{1}{3}$

$m_{AC} = \dfrac{4-1}{0-(-1)} = 3$

b)

these two are opposite reciprocal

so AC is perpendicular to BC

6. (10pts) Let $f(x) = x^2 - 4x$. Find the following (simplify where appropriate).

$f(1) = 1^2 - 4 \cdot 1 = 1 - 4 = -3$

$f(\sqrt{7}) = \sqrt{7}^2 - 4\sqrt{7} = 7 - 4\sqrt{7}$

$f(-a) = (-a)^2 - 4(-a)$
$= a^2 + 4a$

$f(z+5) = (z+5)^2 - 4(z+5)$
$= z^2 + 10z + 25 - 4z - 20$
$= z^2 + 6z + 5$

7. (6pts) Find the domain of the function below and write it using interval notation.

$f(x) = \dfrac{\sqrt{3-2x}}{x+4}$

Must have $3 - 2x \geq 0$

$3 \geq 2x$

$x \leq \dfrac{3}{2}$

Can't have $x + 4 = 0$

$x = -4$

Domain = $(-\infty, -4) \cup (-4, \frac{3}{2}]$

8. (7pts) Solve and write the solution in interval notation.

$2x + 5 < 1$ or $3x - 5 > 8$

$2x < -4 \qquad 3x > 13$

$x < -2 \qquad x > \frac{13}{3}$

$(-\infty, -2) \cup (\frac{13}{3}, \infty)$
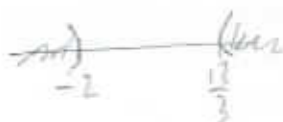
(number line sketch with points at -2 and $\frac{13}{3}$)

9. (8pts) A circle is centered at $(-3, 2)$ and passes through the origin.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.
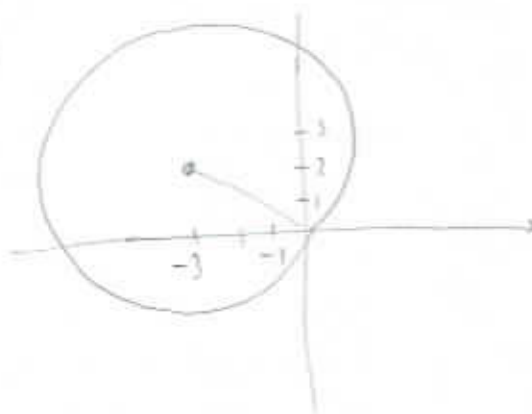
a) $r = $ distance from $(-3, 2)$ to $(0, 0)$

$\quad = \sqrt{(0-(-3))^2 + (0-2)^2}$

$\quad = \sqrt{3^2 + (-2)^2} = \sqrt{5+4} = \sqrt{13}$

$(x - (-3))^2 + (y - 2)^2 = \sqrt{13}^2$

$(x+3)^2 + (y-2)^2 = 13$

Equation of the circle

(circle graph drawn in coordinate plane)

10. (12pts) To solve your plumbing problem, you are considering two specialists. Plumber Hank charges \$20 per visit plus \$60 per hour. Plumber Ricardo charges \$89 per visit, which includes an hour of work, plus \$55 per hour for hours after the first. Assuming your problem requires at least an hour of work time and one visit, for which number of hours is plumber Hank the better option? Solve as an inequality.

$x = $ number of hours used

Hank : $\quad 20 + 60x$

Ricardo : $\quad 89 + 55(x-1)$

Wish to know when

$20 + 60x \leq 89 + 55(x-1)$

$20 + 60x \leq 89 + 55x - 55 \qquad |-55$

$20 + 5x \leq 34 \qquad\qquad |-20$

$5x \leq 14$

$x \leq \frac{14}{5} = 2.8$

For jobs up to 2.8 hours, Hank is better

11. (14pts) Because she was afraid of being late, Emma drove to work at 64 mph. She drove home at 56 mph, so it took her 6 minutes longer.
a) How long did it take Emma to drive to work?
b) How far is her workplace from home?

$$6 \text{ mins} = \frac{1}{10} \text{ hr}$$

dist.	rate	time
d	64	t
d	56	$t + \frac{1}{10}$

a) Emma drove 0.7 hrs $= 42$ min

b) Work place is $0.7 \cdot 64 = 44.8$ miles away from home.

$$d = 64t$$
$$d = 56\left(t + \frac{1}{10}\right)$$

$$64t = 56\left(t + \frac{1}{10}\right)$$
$$64t = 56t + 5.6$$
$$8t = 5.6$$
$$t = \frac{5.6}{8} = 0.7 \text{ hrs}$$

Bonus (10pts) Bill, Rudy and Mike go out for pizza and decide to split the $35.10 bill based on how much each of them ate. Rudy ate half the amount Mike did, and Bill ate four thirds the amount Rudy ate. How much did each pay?

$x =$ amount Mike pays

$\frac{1}{2}x =$ "   Rudy pays

$\frac{4}{3} \cdot \frac{1}{2}x =$ "   Bill pays

$= \frac{2}{3}x$

$$x = \frac{210.6}{13} = 16.2$$

Mike pays $ 16.20
Rudy Pays $ 8.10
Bill pays $ 10.80

$$\frac{2}{3}x + \frac{1}{2}x + x = 35.10 \quad]\cdot 6$$

$$\frac{2}{6} \cdot \frac{2}{3}x + 6 \cdot \frac{1}{2}x + 6x = 210.6$$

$$4x + 3x + 6x = 210.6$$

$$13x = 210.6$$