Calculus 1 — Exam 1
MAT 250, Spring 2024 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -4} f(x) =$

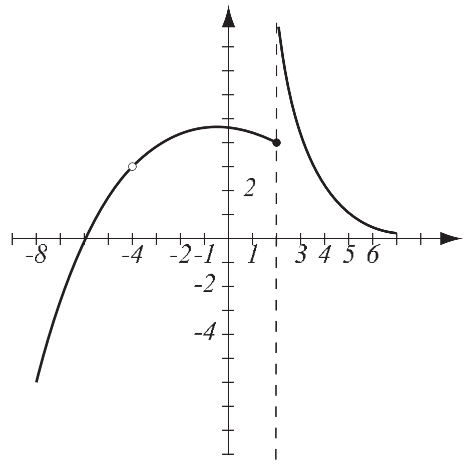
$\lim_{x\to 2-} f(x) =$

$\lim_{x\to 2+} f(x) =$

$\lim_{x\to 2} f(x) =$

$\lim_{x\to \infty} f(x) =$

$\lim_{x\to -\infty} f(x) =$



List points in $(-\infty, \infty)$ where f is not continuous and justify why it is not continuous at those points.

2. (6pts) Let $f(x) = \dfrac{x^2+3}{x+1}$.
a) State the domain of f.
b) Briefly explain why f is continuous on its domain.

3. (10pts) Find $\lim_{x\to 0} x^2\left(3\cos\dfrac{1}{x}+5\right)$. Use the theorem that rhymes with what a forest consists of.

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\lim_{x \to -2} \dfrac{\sqrt{x+3} - 1}{x+2} =$

5. (7pts) $\lim_{x \to \infty} \dfrac{x^2+3}{3x^2 - 5x + 7} =$

6. (5pts) $\lim_{x \to 3} \dfrac{x-3}{x^2 + 4x - 21} =$

7. (6pts) $\lim_{x \to 2^-} \dfrac{x^2+1}{2x-4} =$

8. (7pts) $\lim_{x \to 0} \dfrac{\sin(4x)}{\sin(2x)} =$

9. (14pts) The equation $x = \cos x$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $\left(0, \frac{\pi}{2}\right)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

10. (10pts) Consider the limit $\lim_{x \to 0} \frac{4^x - 1}{x}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values (no more than 5 per table) that will support your answer.

x	$\frac{4^x - 1}{x}$

x	$\frac{4^x - 1}{x}$

11. (12pts) Consider the function defined below. Is the function continuous at point $x = 0$?

$$f(x) = \begin{cases} \dfrac{(1+x)^2 - 1}{x} & \text{if } x < 0 \\ \dfrac{\sin(2x)}{x} & \text{if } x > 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Bonus. (10pts) For each group of properties, draw the graph of the function defined on $[1, 5]$ that satisfies them, if possible. Among the three, one is not possible — explain why.

$f(1) = 4$, $f(5) = 2$ graph of f does not cross line $y = 6$ f is continuous	$f(1) = 4$, $f(5) = 2$ graph of f does not cross line $y = 3$ f is continuous	$f(1) = 4$, $f(5) = 2$ graph of f does not cross line $y = 3$

Calculus 1 — Exam 2
MAT 250, Spring 2024 — D. Ivanšić

Name: ____________________

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(3x^6 - \dfrac{4}{x^4} + \dfrac{5}{\sqrt[6]{x}} + \pi^3\right) =$

2. (5pts) $\dfrac{d}{dx}(x^2+1)\cos x =$

3. (6pts) $\dfrac{d}{du}\dfrac{(u+1)^2}{(u-4)^3} =$

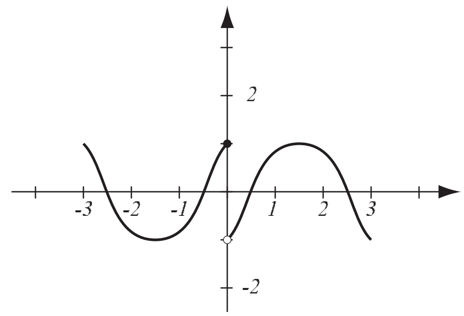
4. (6pts) $\dfrac{d}{d\theta}\dfrac{\cos\theta}{\cos\theta - \sin\theta} =$

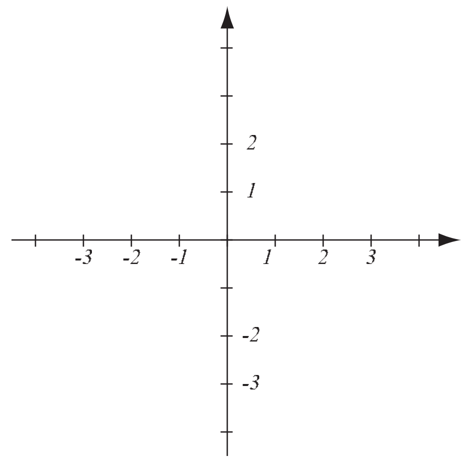
5. (6pts) $\dfrac{d}{dz}\tan\sqrt{\sec z} =$

6. (7pts) Let $y(x) = x^4$.
a) Write the first four derivatives of y.
b) What is the n-th derivative of y for $n \geq 5$?

7. (10pts) The graph of the function $f(x)$ is shown at right.

a) Where is $f(x)$ not differentiable? Why?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.





8. (12pts) Let $f(x) = 2x^2 - 5x + 1$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(2, -1)$.

9. (11pts) Let $g(x) = f(x^3)$ and $h(x) = \dfrac{(f(x))^2}{x}$.

a) Find the general expressions for $g'(x)$ and $h'(x)$.
b) Use the table of values at right to find $g'(1)$ and $h'(3)$.

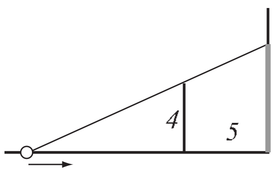
x	1	2	3	4
$f(x)$	3	2	5	-3
$f'(x)$	2	-1	-4	2

10. (6pts) An ball thrown upwards has position (in feet, t in seconds) given by the formula $s(t) = -16t^2 + 40t$.
a) Write the formula for the velocity of the ball at time t.
b) What is the highest altitude that the ball reaches?

11. (11pts) Use implicit differentiation to find y' in general

$\sin(xy) = \sin(x^2) + \sin(y^2)$

12. (14pts) A light source is approaching a 4-meter pole that stands 5 meters in front of a tall wall. If the light source is moving at rate 0.5 meters per second when it is 3 meters from the pole, how fast is the shadow of the pole on the wall growing at that moment? *Hint: similar triangles.*



Bonus. (10pts) Find the points on the curve $y = x^2$ at which the tangent line passes through the point $(2, -1)$, which is not on the curve. *Hint: look for a point (a, a^2) on the curve so that the slope of the line through (a, a^2) and $(2, -1)$ is equal to the slope of the tangent line at (a, a^2).*

Calculus 1 — Exam 3
MAT 250, Spring 2024 — D. Ivanšić

Name: ______________________________

Show all your work!

Differentiate and simplify where appropriate:

1. (4pts) $\dfrac{d}{dx}\, x^2 5^x =$

2. (6pts) $\dfrac{d}{d\theta}\, e^\theta \sin(2\theta) =$

3. (7pts) $\dfrac{d}{du}\, \dfrac{e^u - \sin u}{e^u + \sin u} =$

4. (7pts) $\dfrac{d}{dx}\, \ln \dfrac{\sin^2 x}{\tan x} =$

5. (7pts) $\dfrac{d}{dv}\, \arcsin \sqrt{1 - v^2} =$

6. (9pts) Use logarithmic differentiation to find the derivative of $y = (\cos x)^{\cos x}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim_{x \to \infty} e^{-0.5x} =$

8. (7pts) $\lim_{x \to \infty} (\ln(x+2) - \ln(x^2-1)) =$

9. (7pts) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2} =$

10. (9pts) $\lim_{x \to \infty} x^2 \sin \dfrac{1}{x^2} =$

11. (8pts) $\lim_{x \to \infty} (x^4+3)^{\frac{1}{2x}} =$

12. (11pts) Let $f(x) = \arctan x$.
a) Write the linearization of $f(x)$ at $a = 0$.
b) Use the linearization to estimate $\arctan(-\frac{1}{2})$.
c) In the same coordinate system, draw graphs of the function and the linearization and determine if the estimate in b) is an overestimate or underestimate of $\arctan(-\frac{1}{2})$.

13. (9pts) A cube is measured to have side length of 3 meters, with maximum error 0.5 centimeters. Use differentials to estimate the maximum possible error when computing the surface area of the cube.

14. (7pts) Let $f(x) = x - \sqrt[3]{x}$. Use the theorem on derivatives of inverses to find $(f^{-1})'(6)$.

Bonus. (10pts) Find the limit.

$\lim_{x \to 0+} \ln x \ln(x+1) =$

Calculus 1 — Exam 4
MAT 250, Spring 2024 — D. Ivanšić

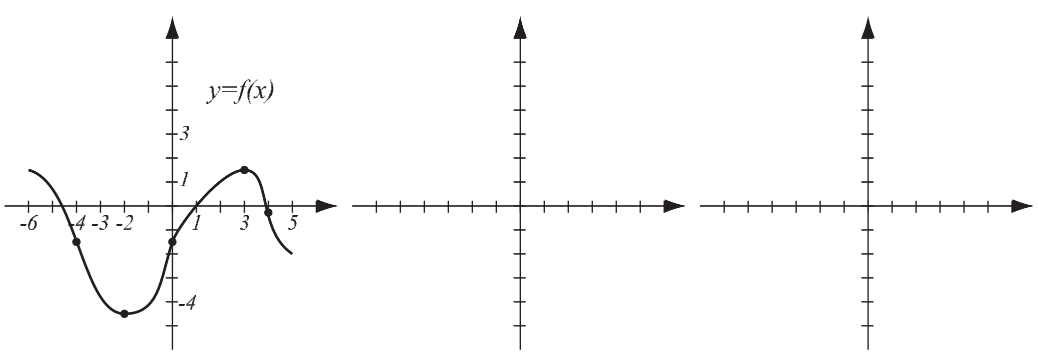
Name:______________________________

Show all your work!

1. (32pts) Let $f(x) = \dfrac{x}{x^2+4}$. The domain of this function is all real numbers (you do not have to verify this). Draw an accurate graph of f by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
d) Use information from a)–c) to sketch the graph.

2. (18pts) Let $f(x) = \sin^2\theta - \cos\theta$. Find the absolute minimum and maximum values of f on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

3. (14pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.



4. (14pts) Consider $f(x) = x^3 - 4x^2$ on the interval $[0, 2]$.
a) Verify that the function satisfies the assumptions of the Mean Value Theorem.
b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. (22pts) Among all rectangles with area 24, find the one with the smallest perimeter.

Bonus. (10pts) Draw the graph of a function that is defined for all real numbers that satisfies:

$f(-1) = -3$, $f(2) = 1$
$f'(x) > 0$ for all x in $(-1, 2)$
$f'(x) < 0$ for all x in $(-\infty, -1)$ and $(2, \infty)$
$f'(-1) = 0$, $f'(2)$ does not exist
$f''(x) > 0$ for all x in $(-\infty, 2)$ and $(2, 3)$
$f''(x) < 0$ for all x in $(3, \infty)$

Calculus 1 — Exam 5
MAT 250, Spring 2024 — D. Ivanšić

Name: ______________________________

Show all your work!

Find the following antiderivatives or definite integrals.

1. (3pts) $\displaystyle\int \frac{1}{\sqrt[4]{x}}\,dx =$

2. (3pts) $\displaystyle\int \sin(2x-\pi)\,dx =$

3. (6pts) $\displaystyle\int (u^2 - 3\sqrt{u})u^3\,du =$

4. (5pts) $\displaystyle\int_0^{\frac{\pi}{4}} 3\sec^2\theta\,d\theta =$

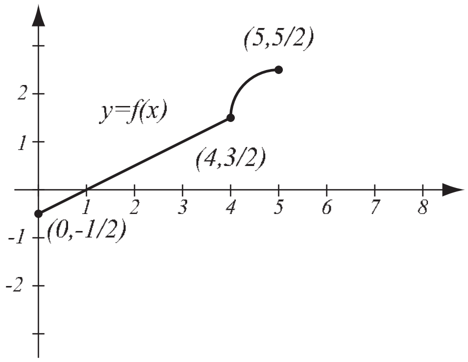
5. (6pts) $\displaystyle\int_{\sqrt{e}}^{e} x - \frac{1}{x}\,dx =$

6. (6pts) Find $f(x)$ if $f'(x) = e^x - \cos x$ and $f(0) = 4$.

7. (15pts) The function $f(x) = x^2 - 2$ is given on the interval $[0, 3]$.
a) Write the Riemann sum M_6 for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

8. (13pts) Find $\int_{-2}^{2} 2x - 2\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (10pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.



$\int_0^4 f(x)\,dx =$

$\int_4^5 f(x)\,dx =$

$\int_0^5 f(x)\,dx =$

10. (16pts) Consider the integral $\displaystyle\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \sin x\,dx.$

a) Use the inequality $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$, where $m \le f(x) \le M$ on $[a,b]$, to give an estimate of the integral. (A graph of $\sin x$ will help you find m and M.)

b) Evaluate the integral and verify your estimate from a).

11. (7pts) Write using sigma notation:

$$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} + \cdots + \frac{13}{256} =$$

12. (10pts) The rate at which temperature in an oven is changing is $\sqrt{t}+2$ degrees Fahrenheit per minute.
a) Use the Net Change Theorem to find how much temperature changed from $t=4$ to $t=9$ minutes.
b) If at time $t=4$ minutes the temperature in the oven was 180°F, what is the temperature at $t=9$ minutes?

Bonus. (10pts) Show that $\sum_{i=1}^{n}(2i-1)=n^2$. (This is $1+3+5+\cdots+(2n-1)=n^2$.) Use either a picture with beads (what is a good way to picture n^2 beads?) that is cleverly divided up, or show it algebraically, for which you may find the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ useful.

Calculus 1 — Final Exam **MAT 250, Spring 2024 — D. Ivanšić**	**Name:** ______________________ *Show all your work!*

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 5+} f(x) =$

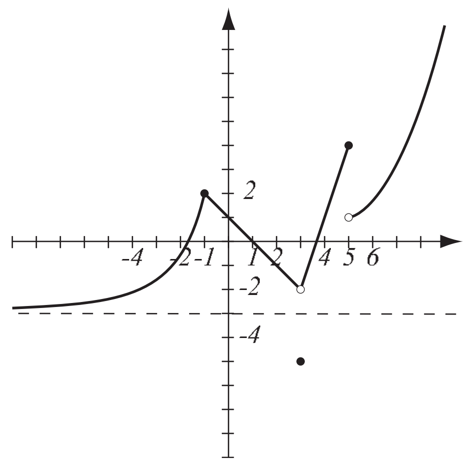
$\lim_{x\to 5-} f(x) =$

$\lim_{x\to 5} f(x) =$

$\lim_{x\to 3} f(x) =$

$\lim_{x\to -\infty} f(x) =$

$\lim_{x\to \infty} f(x) =$



List points where f is not continuous and explain why.

Find the following limits algebraically. Do not use L'Hospital's rule.

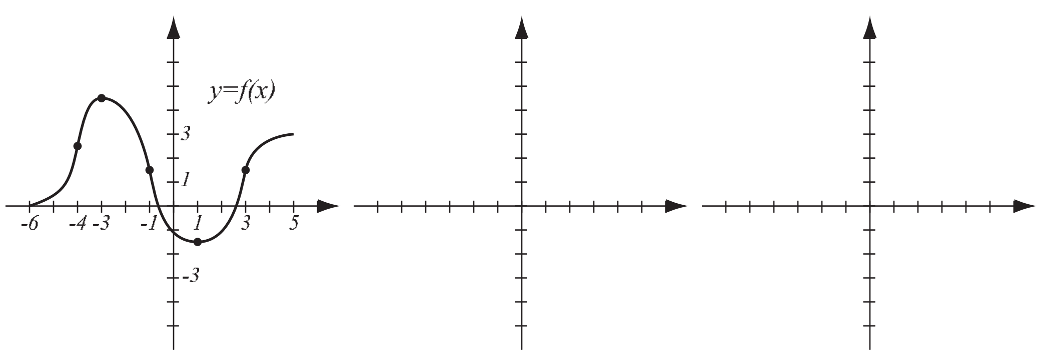
2. (6pts) $\lim_{x\to 3-} \frac{x+1}{2x-6} =$

3. (6pts) $\lim_{x\to \infty} \frac{x^2-15}{4x^2+3x+7} =$

4. (8pts) Find $\lim_{x \to 0+} \sqrt{x}\left(2\sin\frac{1}{x} - 3\right)$. Use the theorem that rhymes with what a forest consists of.

5. (10pts) Write the equation of the tangent line to the curve $y = x^2 e^x$ at point $(-1, \frac{1}{e})$.

6. (12pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.



7. (26pts) Let $f(x) = \dfrac{x}{x^2+1}$. The domain of this function is all real numbers (you do not have to verify this). Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

d) Use information from a)–c) to sketch the graph.

8. (12pts) Let $f(\theta) = \cos^2\theta - \sin\theta$. Find the absolute minimum and maximum values of f on the interval $[0, \frac{3\pi}{2}]$.

9. (6pts) Find $f(x)$ if $f'(x) = 3\sec^2 x + \cos x$ and $f(\frac{\pi}{4}) = -2$.

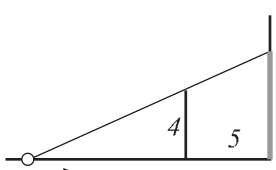
10. (10pts) Consider the integral $\displaystyle\int_0^4 \sqrt{x} - 1\,dx$.

a) Use a picture and the "area" interpretation of the integral to determine whether this integral is positive or negative.

b) Use the Evaluation Theorem to find the integral and verify your conclusion from a).

11. (10pts) The rate at which temperature in an oven is changing is $\dfrac{4}{\sqrt[3]{t}} + 2$ degrees Fahrenheit per minute.
a) Use the Net Change Theorem to find how much temperature changed from $t = 1$ to $t = 8$ minutes.
b) If at time $t = 1$ minutes the temperature in the oven was 170°F, what is the temperature at $t = 8$ minutes?

12. (12pts) A light source is approaching a 4-meter pole that stands 5 meters in front of a tall wall. If the light source is moving at rate 1 meters per second when it is 2 meters from the pole, how fast is the shadow of the pole on the wall growing at that moment? *Hint: similar triangles.*



13. (16pts) Among all rectangles with area 20, find the one with the smallest perimeter.

Bonus. (10pts) Draw the graph of a function that is defined for all real numbers that satisfies:

$f(-1) = -3$, $f(2) = 1$
$f'(x) > 0$ for all x in $(-1, 2)$
$f'(x) < 0$ for all x in $(-\infty, -1)$ and $(2, \infty)$
$f'(-1) = 0$, $f'(2)$ does not exist
$f''(x) > 0$ for all x in $(-\infty, 2)$ and $(2, 3)$
$f''(x) < 0$ for all x in $(3, \infty)$