Calculus 1 — Exam 5
MAT 250, Fall 2023 — D. Ivanšić

Name: _____

Show all your work!

Find the following antiderivatives or definite integrals.

1. (3pts) $\displaystyle\int \sqrt[6]{x^7}\,dx =$

2. (3pts) $\displaystyle\int e^{7x-1}\,dx =$

3. (6pts) $\displaystyle\int \frac{u^2-3u}{\sqrt{u}}\,du =$

4. (5pts) $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx =$

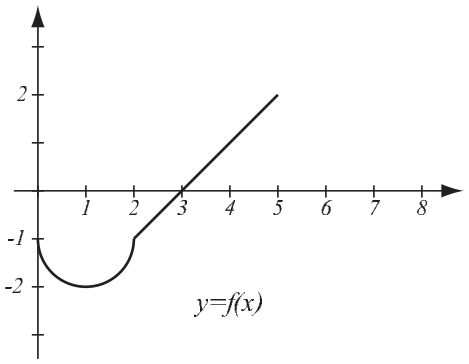
5. (6pts) $\displaystyle\int_0^{\frac{\pi}{3}} \sin\theta + \cos\theta\,d\theta =$

6. (6pts) Find $f(x)$ if $f'(x) = \dfrac{1}{\sqrt{x}} + \dfrac{1}{x}$ and $f(1) = 3$.

7. (15pts) The function $f(x) = \sqrt{x} - 1$ is given on the interval $[0, 3]$.
a) Write the Riemann sum L_6 for this function with six subintervals, taking sample points to be left endpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does L_6 represent?

8. (13pts) Find $\int_{-1}^{3} 4 - 2x\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (10pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.



$\int_0^2 f(x)\,dx =$

$\int_2^5 f(x)\,dx =$

$\int_0^5 f(x)\,dx =$

10. (16pts) Consider the integral $\displaystyle\int_{-1}^{2} -x^2 + 2x\,dx$.

a) Use the inequality $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$, where $m \le f(x) \le M$ on $[a,b]$, to give an estimate of the integral. (A graph of $-x^2+2x$ will help you find m and M.)

b) Evaluate the integral and verify your estimate from a).

11. (7pts) Write using sigma notation:

$$\frac{9}{7} + \frac{16}{9} + \frac{25}{11} + \cdots + \frac{100}{21} =$$

12. (10pts) Helium is pumped into a balloon at rate $e^{-\frac{1}{8}t}$ cubic meters per minute.
a) Use the Net Change Theorem to find how much helium was added from $t = 0$ to $t = 4$ minutes.
b) If at time $t = 0$ there were 2 cubic meters helium in the balloon, how much is there at $t = 4$ minutes?

Bonus. (10pts) A car initially traveling at velocity 20 meters per second accelerates steadily for 5 seconds until it reaches velocity 30 meters per second. Find its position function to help you answer: how far did it travel while it was accelerating?