

Calculus 1 — Exam 3
MAT 250, Fall 2023 — D. Ivanišić

Name: _____
Show all your work!

Differentiate and simplify where appropriate:

1. (4pts) $\frac{d}{dx} 3^{\tan x} =$

2. (6pts) $\frac{d}{du} (u^2 - 2u + 2)e^u =$

3. (7pts) $\frac{d}{du} \frac{e^u + u}{e^u - u} =$

4. (7pts) $\frac{d}{dx} \ln(\sin^2 x \cos^2 x) =$

5. (7pts) $\frac{d}{dt} (t \arccos t - \sqrt{1 - t^2}) =$

6. (9pts) Use logarithmic differentiation to find the derivative of $y = x^{\arctan x}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim_{x \rightarrow 0^+} \ln(2x) =$

8. (6pts) $\lim_{x \rightarrow \infty} e^{-\frac{x^2+1}{x+3}} =$

9. (7pts) $\lim_{x \rightarrow 0} \frac{\cos(4x) - 1}{x^2} =$

10. (9pts) $\lim_{x \rightarrow 0^+} x(\ln x)^2 =$

11. (8pts) $\lim_{x \rightarrow 0^+} (1 - 3x)^{\frac{1}{x}} =$

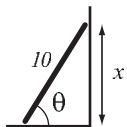
12. (11pts) Let $f(x) = \sqrt{x}$.

a) Write the linearization of $f(x)$ at $a = 4$.

b) Use the linearization to estimate $\sqrt{4.5}$.

c) In the same coordinate system, draw graphs of the function and the linearization and determine if the estimate in b) is an overestimate or underestimate of $\sqrt{4.5}$.

13. (10pts) A 10-foot ladder leans against the wall. Aiming to compute the angle θ that the ladder subtends with the floor, we measure the distance x from the floor to the top of the ladder and find it to be 8 feet with maximum error in measurement $\frac{1}{2}$ inch. Use differentials to estimate the maximum possible error when computing the angle θ . (Since you need to express angle θ as a function of x , an inverse trigonometric function is involved.)



14. (7pts) Let $f(x) = 2^x + x$. Use the theorem on derivatives of inverses to find $(f^{-1})'(11)$.

Bonus. (10pts) Find the derivative and simplify until the bitter end. You will get the derivative of a simpler function. Which one?

$$\frac{d}{dx} \arctan \sqrt{\frac{1-x}{1+x}} =$$