Calculus 1 — Exam 1
MAT 250, Fall 2023 — D. Ivanšić

Name: _____

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -4+} f(x) =$

$\lim_{x\to -4-} f(x) =$

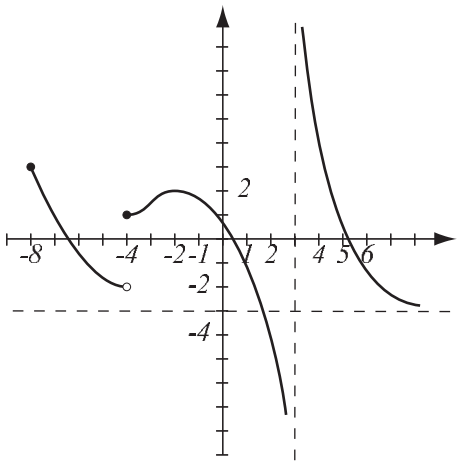
$\lim_{x\to -4} f(x) =$

$\lim_{x\to \infty} f(x) =$

$\lim_{x\to 3-} f(x) =$

$\lim_{x\to 3+} f(x) =$

List points in $[-8, \infty)$ where f is not continuous and justify why it is not continuous at those points.



2. (6pts) Let $\lim_{x\to 1} f(x) = 4$ and $\lim_{x\to 1} g(x) = -3$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 1} \frac{x^3 + g(x)^2}{2 + \sqrt{f(x)}} =$$

3. (10pts) Find $\lim_{x\to 0+} \sqrt{x}\left(3 + \sin\frac{1}{x}\right)$. Use the theorem that rhymes with a vegetable that looks like small green balls.

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\lim_{x\to\infty} \dfrac{x^2-4x+2}{2x+3} =$

5. (5pts) $\lim_{x\to 1} \dfrac{x-1}{x^2+5x-6} =$

6. (7pts) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} =$

7. (6pts) $\lim_{x\to 4^+} \dfrac{x-1}{4-x} =$

8. (7pts) $\lim_{x\to 0} \dfrac{\sin(4x)\tan x}{x^2} =$

9. (14pts) The equation $x^3 - 7 = \sqrt{x}$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 3)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

10. (10pts) Consider the limit $\lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values (no more than 5 per table) that will support your answer.

x	$\dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$

x	$\dfrac{\sqrt{x} - \sqrt{2}}{x - 2}$

11. (12pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(0, 2)$ and $(2, \infty)$
b) For which c is the function continuous at point $x = 2$?

$$f(x) = \begin{cases} x^2 - cx, & \text{if } 0 < x < 2 \\ \dfrac{c}{x} + 5, & \text{if } x \geq 2. \end{cases}$$

Bonus. (10pts) Find the limit algebraically. Do not use the calculator.

$$\lim_{h \to 0} \frac{(2+h)^4 - 16}{h} =$$