| **Calculus 1 — Exam 1** |
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| **MAT 250, Spring 2024 — D. Ivanšić** |

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*Show all your work!*

**1.** (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -4} f(x) =$ 3

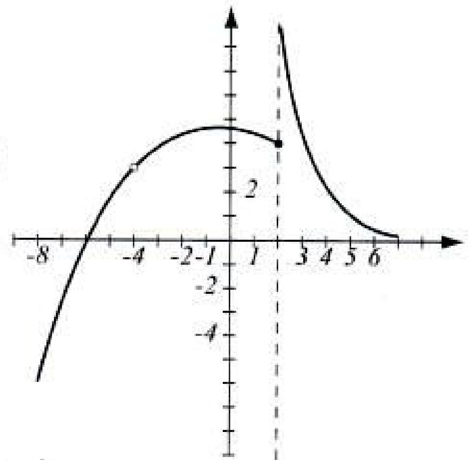
$\lim_{x\to 2-} f(x) =$ 4

$\lim_{x\to 2+} f(x) =$ $\infty$

$\lim_{x\to 2} f(x) =$ DNE, one-sided limits are not equal

$\lim_{x\to \infty} f(x) =$ 0

$\lim_{x\to -\infty} f(x) =$ $-\infty$




List points in $(-\infty,\infty)$ where $f$ is not continuous and justify why it is not continuous at those points.

$x=-4$, $f(-4)$ not defined

$x=2$ $\lim_{x\to 2} f(x)$ DNE

**2.** (6pts) Let $f(x)=\dfrac{x^2+3}{x+1}$.
a) State the domain of $f$.
b) Briefly explain why $f$ is continuous on its domain.

a) Can't have $x+1=0$, $x=-1$. Domain $=(-\infty,-1)\cup(-1,\infty)$

b) $x^2+3$ and $x+1$ are polynomials, so continuous on $\mathbb{R}$
Then its quotient is continuous as long as denominator is not zero, that is, for $x\neq -1$.

**3.** (10pts) Find $\lim_{x\to 0} x^2\left(3\cos\dfrac{1}{x}+5\right)$. Use the theorem that rhymes with what a forest consists of.

$-1 \le \cos\frac{1}{x} \le 1$

$-3 \le 3\cos\frac{1}{x} \le 3$

$2 \le 3\cos\frac{1}{x}+5 \le 8$

$2x^2 \le x^2\left(3\cos\frac{1}{x}+5\right) \le 8x^2$

$\lim_{x\to 0} 2x^2=0$ $\lim_{x\to 0} 8x^2=0$

These are equal, so by the squeeze theorem, $\lim_{x\to 0} x^2\left(3\cos\frac{1}{x}+5\right)=0$

Find the following limits algebraically. Do not use the calculator.

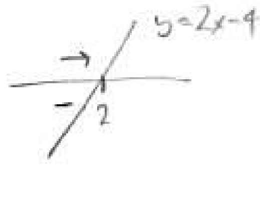
**4.** (7pts) $\lim_{x \to -2} \frac{\sqrt{x+3}-1}{x+2} = \lim_{x\to -2} \frac{\sqrt{x+3}-1}{x+2} \cdot \frac{\sqrt{x+3}+1}{\sqrt{x+3}+1} = \lim_{x\to -2} \frac{\sqrt{x+3}^2 - 1^2}{(x+2)(\sqrt{x+3}+1)}$

$= \lim_{x\to -2} \frac{\overbrace{x+3-1}^{=x+2}}{(\cancel{x+2})(\sqrt{x+3}+1)} = \lim_{x\to -2} \frac{1}{\sqrt{x+3}+1} = \frac{1}{\sqrt{-2+3}+1} = \frac{1}{2}$

**5.** (7pts) $\lim_{x \to \infty} \frac{x^2+3}{3x^2-5x+7} = \lim_{x\to\infty} \frac{\cancel{x^2}\left(1+\underbrace{\frac{3}{x^2}}_{\to 0}\right)}{\cancel{x^2}\left(3-\underbrace{\frac{5}{x}}_{\to 0}+\underbrace{\frac{7}{x^2}}_{\to 0}\right)} = \frac{1+0}{3-0+0} = \frac{1}{3}$

**6.** (5pts) $\lim_{x \to 3} \frac{x-3}{x^2+4x-21} = \lim_{x\to 3} \frac{(\cancel{x-3})}{(\cancel{x-3})(x+7)} = \lim_{x\to 3} \frac{1}{x+7} = \frac{1}{10}$

**7.** (6pts) $\lim_{x \to 2^-} \frac{x^2+1}{2x-4} = \frac{4+1}{4-4} = \frac{5}{0^-} = 5 \cdot \frac{1}{0^-} = 5(-\infty) = -\infty$




**8.** (7pts) $\lim_{x \to 0} \frac{\sin(4x)}{\sin(2x)} = \lim_{x\to 0} \frac{\frac{\sin(4x)}{4x} \cdot 4\cancel{x}}{\frac{\sin(2x)}{2x} \cdot 2\cancel{x}} = \frac{1 \cdot 4}{1 \cdot 2} = 2$

**9.** (14pts) The equation $x = \cos x$ is given. $x - \cos x = 0$
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, \frac{\pi}{2})$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

a) Let $f(x) = x - \cos x$ - it is continuous

$f(0) = 0 - \cos 0 = -1$
$f(\frac{\pi}{2}) = \frac{\pi}{2} - \cos\frac{\pi}{2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Since $f(0) < 0 < f(\frac{\pi}{2})$, by IVT there is a c in $(0, \frac{\pi}{2})$ s.t. $f(c) = 0$. c is then a solution of the equation

b) Using calculator, we find

$f(0.73) = -0.0152$
$f(0.74) = 0.00153$

Since $f(0.73) < 0 < f(0.74)$
By IVT there is a c in $(0.73, 0.74)$ s.t. $f(c) = 0$

**10.** (10pts) Consider the limit $\lim_{x\to 0} \dfrac{4^x - 1}{x}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values (no more than 5 per table) that will support your answer.

| $x$ | $\frac{4^x-1}{x}$ |
|---|---|
| 0.1 | 1.486984 |
| 0.01 | 1.395948 |
| 0.001 | 1.387256 |
| $10^{-4}$ | 1.386390 |
| $10^{-5}$ | 1.386304 |

| $x$ | $\frac{4^x-1}{x}$ |
|---|---|
| $-0.1$ | 1.294494 |
| $-0.01$ | 1.376730 |
| $-0.001$ | 1.385334 |
| $-10^{-4}$ | 1.386198 |
| $-10^{-5}$ | 1.386285 |

It appears that

$\lim_{x\to 0} \frac{4^x-1}{x} = 1.386$

**11.** (12pts) Consider the function defined below.
a) Explain why the function is continuous on intervals $(-\infty, 0)$ and $(0, \infty)$
b) Is the function continuous at point $x = 0$?

$$f(x) = \begin{cases} \dfrac{(1+x)^2 - 1}{x} & \text{if } x < 0 \\ \dfrac{\sin(2x)}{x} & \text{if } x > 0 \\ 2 & \text{if } x = 0 \end{cases}$$

a) On $(-\infty,0)$ and $(0,\infty)$ the function is given by a single formula, so continuous

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0-} \frac{(1+x)^2-1}{x} = \lim_{x\to 0-} \frac{1+2x+x^2-1}{x} = \lim_{x\to 0-} \frac{2x+x^2}{x}$$

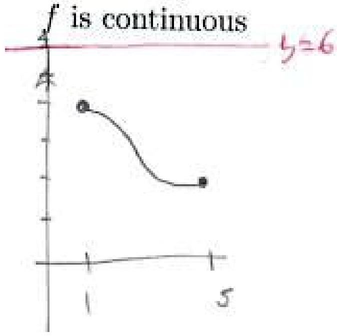
$$= \lim_{x\to 0-} \frac{\cancel{x}(2+x)}{\cancel{x}} = 2+0 = 2$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} \frac{\sin(2x)}{x} = \lim_{x\to 0+} \frac{2\sin(2x)}{2x} = 2\cdot 1 = 2$$

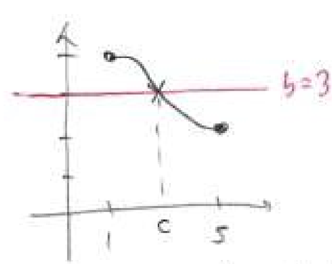
So $\lim_{x\to 0} f(x) = 2$, and $f(0) = 2$, so $f$ is continuous at $x=0$

**Bonus.** (10pts) For each group of properties, draw the graph of the function defined on $[1, 5]$ that satisfies them, if possible. Among the three, one is not possible — explain why.

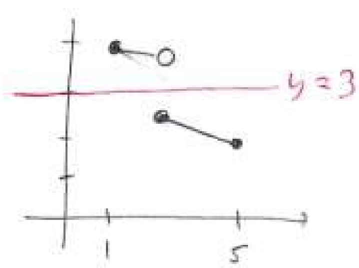
$f(1) = 4$, $f(5) = 2$
graph of $f$ does not cross line $y = 6$
$f$ is continuous




$f(1) = 4$, $f(5) = 2$
graph of $f$ does not cross line $y = 3$
$f$ is continuous




$f(1) = 4$, $f(5) = 2$
graph of $f$ does not cross line $y = 3$




Not possible due to IVT:
Since $f(5) < 3 < f(1)$, there is a
$c$ in $(1,5)$ so that $f(c) = 3$, so graph crosses line $y=3$.