

College Algebra — Joysheet 1
MAT 140, Spring 2024 — D. Ivanšić

Name: _____

Covers: JIT 6, 7, 12, 14, 15, 17

Show all your work!

Write interval notation and sketch on the number line.

1. (3pts) $\{x|x \geq 3\}$

2. (3pts) $\{x|-1 \leq x < 6\}$

Solve the equations.

3. (3pts) $2x - 1 = 4x - 5$

4. (4pts) $3(2t - 3) + 1 = 2(1 - t) - 4$

Simplify and write in standard form:

5. (4pts) $(x + 3)^2 - 2x^2(x - 7) =$

6. (4pts) $(x - 3)(x + 9) + 3x(x - 4) =$

Simplify and write the answer so all exponents are positive:

7. (2pts) $b^2(4b)^3 =$

8. (2pts) $\frac{(2u)^4}{u^3} =$

9. (3pts) $a^8(a^{-2}b^4)^5 b^{-11} =$

10. (5pts) $(u^{-2}v^{-4})^3(4u^{-3}v^5)^2 =$

11. (7pts) $\frac{(6x^{-5}y^3)^2}{(2x^4y^{-2})^3} =$

Factor the following.

12. (4pts) $x^2 - 5x - 14 =$

13. (4pts) $x^2 + 14x - 32 =$

Use the *ac*-method or another method to factor. Show how you got your answer.

14. (6pts) $2x^2 - 5x - 12 =$

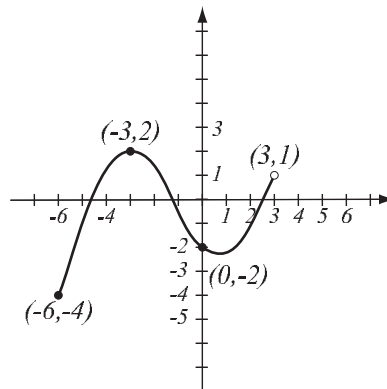
15. (6pts) $6x^2 + 7x - 10 =$

1. (11pts) Draw the triangle with vertices $A = (-2, 1)$, $B = (0, -2)$ and $C = (4, 2)$ in the coordinate plane.
- Compute the lengths of all sides of the triangle and find the perimeter of the triangle.
 - Find the distances from the vertices to the origin.
 - Which number is bigger, the perimeter of the triangle or the sum of distances to the origin?

2. (10pts) Find the equation of the circle, if its diameter has endpoints $(-3, 2)$ and $(1, 0)$. Draw the circle.

3. (8pts) Use the graph of the function f at right to answer the following questions.

- Find $f(-3)$ and $f(3)$.
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 1$?



4. (12pts) The function $f(x) = x^2 - 7\sqrt{x+3}$ is given.

a) Use your calculator to accurately draw its graph. Draw the graph here, and indicate units on the axes.

b) Find all the x - and y -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (9pts) Find the domain of each function and write it using interval notation.

$$f(x) = \sqrt{x} - \frac{3x+1}{4x-7}$$

$$g(x) = \frac{\sqrt[3]{x}}{x-9}$$

6. (10pts) Let $h(x) = \frac{3\sqrt{x}}{x^2 - x - 6}$. Find the following (simplify where appropriate).

$$h(4) =$$

$$h(-2) =$$

$$h(a^2) =$$

$$h(x-2) =$$

1. (5pts) Find the equation of the line (in form $y = mx + b$) that passes through $(5, 2)$ whose y -intercept is 4. Draw the graph of the line.

2. (10pts) Find the equation of the line (in form $y = mx + b$) passing through $(-1, -3)$ that is perpendicular to the line $x - 4y = 8$. Draw both lines.

3. (9pts) Consider two triangles inscribed into the circle $x^2 + y^2 = 25$. Both triangles have vertices $A = (-5, 0)$ and $B = (5, 0)$. The third vertex is either $C = (0, 5)$ or $D = (-3, 4)$.

- Draw the circle and the triangles ABC and ABD .
- Find the slopes of the sides of the two triangles.
- Use slopes to verify that both triangles ABC and ABD are right triangles.

4. (4pts) According to government data, the price (on average) of a pound of ground beef was \$3.55 in December of 2016, and \$5.21 in December of 2023. What is the average rate of change of the price of a pound of ground beef from 2016 to 2023? (Time is measured in years.) What are the units for the average rate of change?

5. (12pts) The electric bill for a household was \$123.87 in a month when it used 764 kWh (kilowatt-hours) of electricity. In another month, it used 1210 kWh and was billed \$181.85.
- Assuming that electricity cost $C(x)$ is a linear function of the amount of electricity x used (in kWh), write a formula for $C(x)$.
 - What is the cost if no electricity is used during a month? What is the meaning of this number?
 - What is the meaning of the slope in this example?

6. (20pts) While, as in the previous problem, electricity cost is generally a linear function of usage, in real life this is not so exact, owing to varying per-kWh costs over the year. Below is data for electricity cost C and usage x (in kWh) for a household for months April to September. Solve the problems below with accuracy 6 decimal points.
- Draw the scatterplot of the data. Does the relationship look linear?
 - Use two points in the scatterplot to get an equation of a line that models the relationship between x and C . Draw the line on the graph.
 - Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.
 - Find coefficient of correlation r . How strong is the linear relationship between x and C ?
 - If this household uses 750 kWh of electricity, how much do you expect the cost to be?

x	C
905	96.69
573	67.67
215	34.33
433	56.96
826	96.99
817	93.50

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $-4 \leq 2 - 7x < 2$

2. (7pts) $3x - 1 < -2$ or $5x + 2 > 13$

3. (6pts) Find the domain of the function in interval notation: $f(x) = \frac{\sqrt{5 - 2x}}{x + 1}$.

4. (14pts) Two lawyers charge for their services in the following way: Dewey charges \$700 for the first three hours, and then \$180 per hour for the hours after the first three. Cheatham charges \$600 for the first four hours, and then \$200 per hour for the hours after the first four. If you need at least four hours of legal work, for which number of hours is it preferable to hire Dewey? Solve as an inequality.

5. (14pts) Sophie traveled to Louisville at 72 mph. Returning, she took the scenic route and averaged 62 mph. The way back was 45 miles longer and took 1.5 hours longer to drive than the way to Louisville.

a) How long did it take to drive to Louisville?

b) How long is the scenic route?

6. (14pts) Diego invested some money at 3% simple interest, and some at 4% simple interest. If his total interest over 1 year was \$80.40, and he invested \$400 more at 4% than he did at 3%, how much did he invest at each interest rate?

1. (10pts) Use your calculator to accurately sketch the graph of the function

$f(x) = \frac{x^2 + 3x + 9}{x^2 - 3x + 4}$. (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \frac{1}{x^2 - 1}$, $g(x) = \sqrt{x + 3}$. Find the following (simplify where possible):

$$(f - g)(6) =$$

$$(fg)(-2) =$$

$$\frac{g}{f}(x) =$$

$$(g \circ f)(3) =$$

$$(f \circ g)(x) =$$

The domain of $(fg)(x)$ in interval notation

3. (8pts) Consider the function $h(x) = \sqrt{7x^2 + 2}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

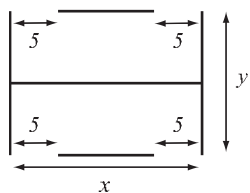
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} \frac{1}{2}x + 3, & \text{if } -4 \leq x < 2 \\ x - 4 & \text{if } 2 \leq x < 7. \end{cases}$$

5. (14pts) A farmer is building a 1000-square-foot barn with two separated areas, each with two doors of width 5 feet. The farmer wishes to minimize the construction cost, which is same as minimizing the total length of the walls.

a) Express the total length of the walls as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the barn for which the total length of the walls is minimal? What is the minimal wall length?



1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^2 + 7x - 3$$

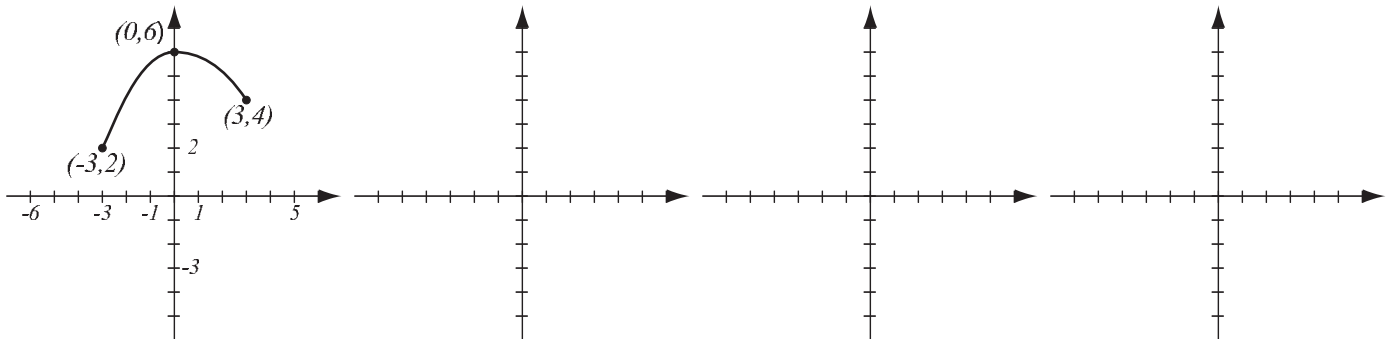
$$g(x) = x^2 - 4|x| + 5$$

$$h(x) = x^5 - 8x^3 + 20x$$

2. (16pts) Draw the graphs of $f(x) = -(x - 2)^3$ and $g(x) = 3 - 4|x|$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of $y = \frac{1}{x}$, shifted left 2 units,
 - shape of $y = x$, reflected about the x -axis, then shifted up 3
 - shape of $y = \sqrt[3]{x}$, reflected about the y -axis, then stretched vertically by factor 3, then stretched horizontally by factor 2.

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) - 3$, $\frac{1}{2}f(x - 1)$ and $-f(\frac{3}{2}x)$ and label all the relevant points.



College Algebra — Joysheet 7
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Name: _____

Covers: JIT 13, 21–23, 25

Show all your work!

Use formulas to expand:

1. (4pts) $(4x + y)^2 =$

2. (5pts) $(a^3 - 5b)^2 =$

3. (4pts) $(2x - y^4)(2x + y^4) =$

4. (8pts) Compute expressions with fractions by hand.

$$\frac{5}{3} \cdot \frac{18}{25} =$$

$$\frac{7}{6} \div \frac{98}{9} =$$

$$\frac{2}{3} + \frac{5}{9} =$$

$$\frac{3}{10} - \frac{7}{18} =$$

Multiply or divide the rational expressions.

5. (7pts) $\frac{3x^2 - 12x + 9}{6x + 24} \cdot \frac{x + 4}{x^2 - 1} =$

6. (7pts) $\frac{3x^2 - 11x - 4}{6x + 2} \div \frac{x^2 - 2x - 8}{3x + 6} =$

Add or subtract the rational expressions.

7. (6pts) $\frac{2x - 3}{x - 2} - \frac{x^2 - 7}{x^2 - 7x + 10} =$

8. (8pts) $\frac{x - 1}{x^2 - 3x - 28} + \frac{3x + 1}{2x^2 + 3x - 20} =$

Simplify the following expressions, assuming all variables are positive.

9. (3pts) $\sqrt{27x^6y^3} =$

10. (4pts) $\sqrt[3]{4u^2v^5} \sqrt[3]{10u^2v} =$

11. (4pts) $(7 + 2\sqrt{3})(\sqrt{75} - 2) =$

College Algebra — Joysheet 8
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Name: _____

Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $7i(3 - 2i) + (3i)^2 =$

2. (6pts) $\frac{4 - 7i}{3 - i} =$

3. (4pts) Simplify and justify your answer.

$i^{213} =$

4. (8pts) The number of barrels of oil (in thousands) in storage at a refinery is described by the function $N(x) = -x^2 + 14x + 31$, where x is the number of days after March 23rd.

a) On what dates did the refinery have 55 thousand barrels in storage?

b) On what date did the number of barrels of oil reach its maximum?

5. (8pts) Solve the equation: $x^4 + 16x^2 + 63 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 8x - 15 = 0$

7. (12pts) The quadratic function $f(x) = -x^2 + 4x - 8$ is given. Do the following without using the calculator.

- a) Find the x -intercepts of its graph, if any. Find the y -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

8. (12pts) In a rectangular plot of land, the length is 5 meters more than the width. If the length and width are each increased by 2 meters, we get a rectangular plot whose area is twice the area of the original plot. What are the dimensions of the original plot?

College Algebra — Joysheet 9
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Name: _____

Covers: 3.3, 3.4, 3.5 Show all your work!

1. (4pts) Solve the equation.

$$|3x - 7| = 5$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x + 7| > 5$$

$$|5x - 3| < 10$$

Solve the equations:

3. (8pts) $\frac{x+1}{x-7} + \frac{2x-1}{x+2} = \frac{3x+51}{x^2-5x-14}$

4. (8pts) $2x + 3 = x - \sqrt{5x + 21}$

5. (14pts) Mr. Beast ejects upwards an old-style TV set from a deck at height 15 meters with initial velocity 42 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 42t + 15$.

a) Sketch the graph of the height function.

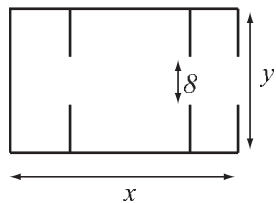
b) When does the TV reach its greatest height, and what is that height?

c) When does the TV fall to the ground? Will it make for good video?

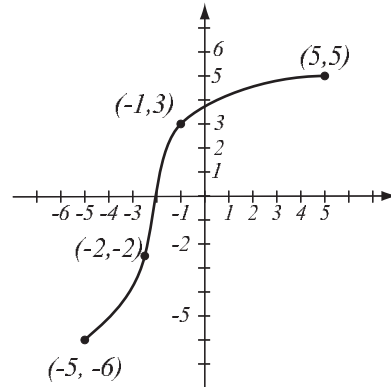
6. (14pts) Chantelle is building a small gallery with three rooms that have doors 8 feet wide. She has budgeted for 1200 ft of walls and wishes to maximize the area of the gallery.

a) Express the total area of the gallery as a function of one of the sides of the rectangle. What is the domain of this function?

b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the gallery that has the greatest total area? What is the greatest total area possible?



1. (6pts) The graph of a function f is given.
- Is this function one-to-one? Justify.
 - If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points.



2. (12pts) Let $f(x) = \frac{x+3}{x-1}$. Find the formula for f^{-1} . Find the ranges of f and f^{-1} .

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_3 81 = \quad \log_2 \frac{1}{32} = \quad \log_{64} 8 = \quad \log_{b^2} b^8 =$$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_{30} 5$ with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$3,000 and annual interest rate of 5.22%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of $f(x) = \ln(3x + 7)$.

7. (8pts) In a certain rectangle, the length and width are related by the equation $l = \frac{40}{w - 1}$.

- Find length if width is 5 meters and 2 meters .
- Find a formula for the inverse function and explain what it represents.
- What widths correspond to lengths 8 meters and 4 meters?

8. (7pts) Using transformations, draw the graph of $f(x) = -\ln(x - 1)$. Explain how you transform the graph of a basic function in order to get the graph of f . Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If $\log_a 3 = 0.5646$ and $\log_a 4 = 0.7124$, calculate:

$$\log_a 48 =$$

$$\log_a \frac{4}{3} =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_9 (729x^2y^5) =$$

$$\ln \frac{\sqrt[7]{x^3y^4}}{x^3z^2\sqrt{e}} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$2\log_4(6x^2) - 3\log_4 x^2 - 3\log_4(2y^4) =$$

$$2\ln(x^2 - 3x - 10) + \ln(x - 5) - 3\ln(x + 2) =$$

4. (3pts) Simplify. $\log_2 2^{x^2-3x} =$ $10^{\log(5-4x)} =$

Solve the equations.

5. (5pts) $2^{x-5} = \left(\frac{1}{8}\right)^{2x}$

6. (7pts) $5^{2x-1} = 7^{x-4}$

7. (5pts) Charles buys shares in a mutual fund for \$9,000. The shares' value increases by 12% every year, so after t years the value of Charles' investment is given by the function $V(t) = 9000 \cdot (1.12)^t$. When will the shares in the mutual fund be worth \$20,000?

8. (12pts) According to census data, Austin, TX, had 790,000 inhabitants in 2010 and 962,000 in 2020. Assume the population of Austin grows exponentially.

a) Write the function describing the number $P(t)$ of people in Austin t years after 2010. Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 1,500,000?