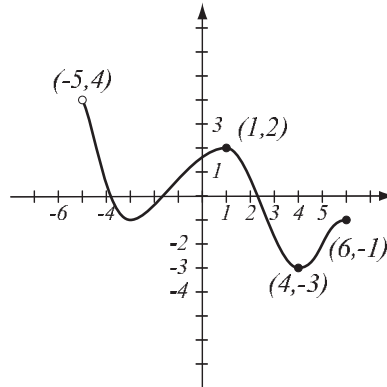


1. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- Find:  $f(1) =$        $f(4) =$
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = -2$ ?



2. (12pts) Use your calculator to accurately sketch the graph of  $f(x) = x^3 - 5x^2 + x - 2$ .

- Draw the graph on paper and indicate units on the axes.
- Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).
- State the range of the function in interval notation.

3. (5pts) Find the equation of the line (in form  $y = mx + b$ ) that is parallel to the line  $y = 3x + 2$  and passes through the point  $(1, -3)$ . Draw the requested line.

4. (10pts) Find the equation of the line (in form  $y = mx + b$ ) that is perpendicular to the line  $2x - 3y = 9$  and contains the point  $(1, 4)$ . Draw both lines.

5. (8pts) In a coordinate system, draw the quadrangle with vertices  $A = (0, -2)$ ,  $B = (5, -4)$ ,  $C = (1, 2)$  and  $D = (-2, 1)$ .
- a) Compute the slopes of the sides.
- b) Use slopes determine if this is a trapezoid (a quadrangle with two sides parallel).

6. (10pts) Let  $f(x) = \frac{x^2 + 1}{2x - 1}$ . Find the following (simplify where appropriate).

$$f(2) =$$

$$f\left(\frac{1}{2}\right) =$$

$$f(\sqrt{t}) =$$

$$f(u + 3) =$$

7. (6pts) Find the domain of the function below and write it using interval notation.

$$f(x) = \frac{\sqrt{5 - 2x}}{2x + 4}$$

8. (5pts) Solve and write the solution in interval notation.

$$2 \leq 5x - 3 < 5$$

9. (10pts) The endpoints of a diameter of a circle are  $(-3, 2)$  and  $(1, -4)$ .

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

10. (12pts) At her coffee shop job, Esperanza can be paid in one of these ways:

A) Hourly salary of \$13.50.

B) Flat pay of \$99 for the first 10 hours, plus \$15 an hour for hours past 10.

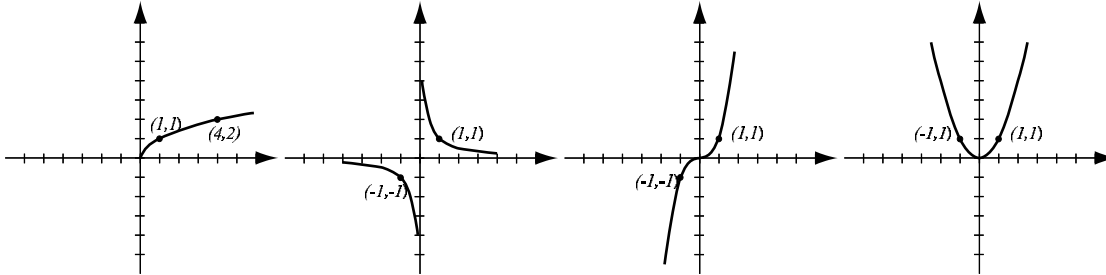
Assuming Esperanza always works at least 10 hours per week, for which number of hours worked is pay plan A better? Solve as an inequality.

**11.** (14pts) Kurt drives to Hopkinsville on the highway at 70mph. Due to flooding of the highway, on the way back he has to take a slower route where he averages 58mph. This route is 20 miles longer and takes him 30 minutes more to drive.

- a) How long did Kurt drive to Hopkinsville?
- b) How long was the slower route?

**Bonus** (10pts) Find a point  $(0, s)$  on the  $y$ -axis that has the same distance to  $(4, 0)$  and  $(1, 3)$ . Draw a picture. *Hint: use the distance formula to set up the equation in  $s$  that says those distances are same. Then rid the equation of square roots by squaring it, and solve it.*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let  $f(x) = \frac{1}{\sqrt{5-x}}$ ,  $g(x) = \sqrt{x+4}$ .

Find the following (simplify where possible):

$$(f - g)(0) =$$

$$(fg)(1) =$$

$$\frac{f}{g}(x) =$$

$$(g \circ f)(-4) =$$

$$(f \circ g)(x) =$$

The domain of  $fg$  in interval notation

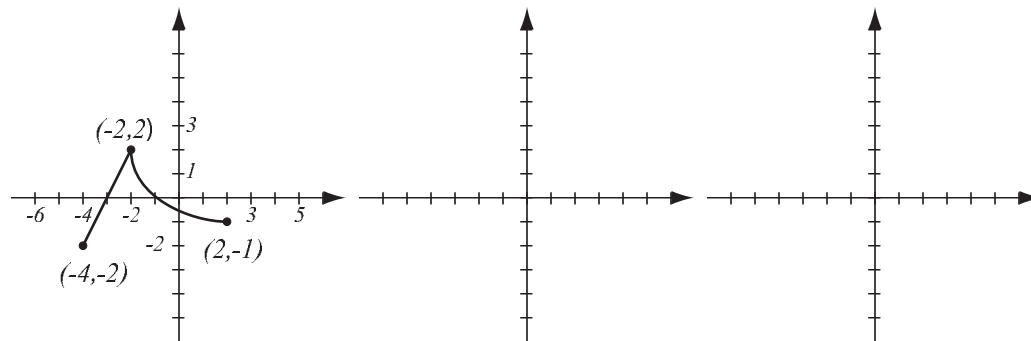
3. (6pts) Consider the function  $h(x) = \frac{1}{x^2 - 7}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:

a) shape of  $y = \sqrt{x}$ , stretched vertically by factor 4.

b) shape of  $y = |x|$ , then reflected over the  $x$ -axis and then shifted up 3.

5. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $-f(x - 3)$  and  $f(2x) + 1$  and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -\frac{1}{2}x + 4, & \text{if } x \leq 6 \\ x - 3, & \text{if } x > 6 \end{cases}$$

7. (8pts) In three separate coordinate systems, sketch a graph of a function that is even, odd or neither. You can draw any curve you like, as long as it has the property requested.

even

odd

neither

8. (20pts) Let  $f(x) = \frac{20x}{x^2 + 5}$  (answer with 6 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of  $f$  (on paper!). (If you cannot enter the function in fraction form in the calculator, make sure you put parentheses around the denominator.) Indicate units on the axes.

b) Determine algebraically whether the function is odd, even, or neither.

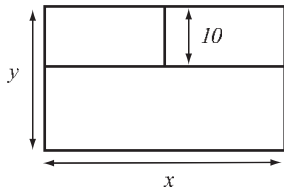
c) Verify your conclusion from b) by stating symmetry.

d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.

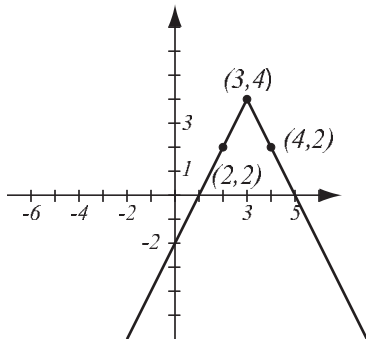
e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) On a beach, a rectangular section of the sea with area 4000 square meters is to be roped off with a float line and divided into three parts, as in the picture, with smaller parts exactly 10 meters wide. To minimize cost, the total length of float line has to be as small as possible.

- Express the total length of float line as a function of the length of one of the sides  $x$ . What is the domain of this function?
- Graph the function in order to find the minimum. What are the dimensions of the section that uses the least length of float line? What is the smallest total length of float line?



**Bonus.** (10pts) The graph below was obtained by transformations of a graph of a standard function. Identify the standard function and the transformations and use them to write the formula for the function in the picture.





College Algebra — Exam 3  
MAT 140C, Spring 2024 — D. Ivanšić

Name: \_\_\_\_\_  
*Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (5pts)  $2i(1 - 3i)^2 =$

2. (5pts)  $\frac{3 + 2i}{-2 - i} =$

3. (4pts) Simplify and justify your answer.

$i^{111} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 14x = 15$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|3x + 2| \leq 4$

6. (6pts) Let  $P(x)$  be a polynomial of degree 5.

a) Draw a graph of  $P$  that has the maximal number of turning points.

b) Explain why a  $P$  with 4  $x$ -intercepts and 2 turning points is not possible.

7. (12pts) The quadratic function  $f(x) = -x^2 - 3x + 18$  is given. Do the following without using the calculator.

a) Find the  $x$ - and  $y$ -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

Solve the equations:

8. (8pts)  $\frac{2}{x} = \frac{x+1}{x-4} + \frac{2x-28}{x^2-4x}$

9. (8pts)  $x + \sqrt{5x-26} = 4$

**10.** (14pts) The polynomial  $f(x) = (x + 2)(x - 3)^2(1 - x)$  is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the  $y$ -intercept.

c) Use the graphing calculator along with a) and b) to accurately sketch the graph of  $f$  (yes, on paper!).

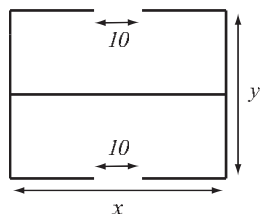
d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

**11.** (12pts) The combined area of two squares is  $84 \text{ cm}^2$ . If the larger square has sides 2 cm longer than the smaller square, what are the lengths of the sides of the two squares?

**12.** (14pts) Jeffrey is designing a combo gas station / fast food restaurant building with 10-foot wide entrances on either side. His budget allows for 350 feet of total wall length. Jeffrey's goal is to maximize the total area of the building.

a) Express the total area of the building as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible total area, and what is the biggest possible total area?



**Bonus.** (10pts) Find the equation of the parabola that goes through the points  $(-2, 21)$ ,  $(0, 7)$  and  $(2, 9)$ . *Hint: you are looking for an equation of form  $y = ax^2 + bx + c$  with unknown  $a$ ,  $b$  and  $c$ . Use the three listed points to get equations for  $a$ ,  $b$  and  $c$  and solve them.*

College Algebra — Exam 4  
MAT 140C, Spring 2024 — D. Ivanšić

Name: \_\_\_\_\_  
*Show all your work!*

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_3 9 = \qquad \log_4 \frac{1}{64} = \qquad \log_b \sqrt[3]{b^5} = \qquad \log_{\sqrt{a}} a^3 =$$

2. (4pts) Use the change-of-base formula and your calculator to find  $\log_7 0.07$  with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If  $\log_a 2 = 0.3155$  and  $\log_a 5 = 0.7325$ , calculate the following values:

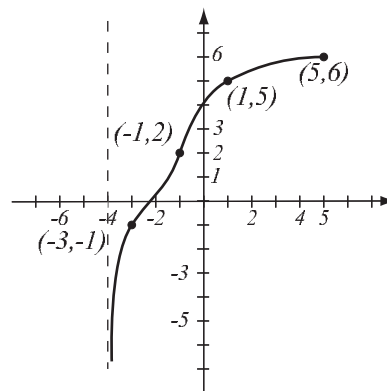
$$\log_a \frac{5}{2} = \qquad \log_a 100 =$$

4. (4pts) Simplify.

$$\log_4 4^{1-3x} = \qquad e^{\ln(2a-3b)} =$$

5. (8pts) If you deposit \$4,000 in an account bearing 3.4% interest, compounded daily, how much is in the account after 5 years?

6. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points, and showing any asymptotes.



7. (9pts) Let  $f(x) = \frac{2x - 5}{x + 6}$ .
- Find the formula for  $f^{-1}$ .
  - Find the range of  $f$ .

8. (6pts) Using transformations, draw the graph of  $f(x) = 3^{x-2} + 1$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Indicate at least one point on the graph and any asymptotes.

**9.** (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \left( 16\sqrt[4]{x^7}y^3 \right) =$$

$$\log_5 \frac{125x^7y^4}{y^7} =$$

**10.** (12pts) Write as a single logarithm. Simplify if possible.

$$2\log_6(s^7t^4) + 3\log_6(s^{-4}t^3) =$$

$$2\log(x^2 - x - 20) + \log(x - 5) - 3\log_2(x + 4) =$$

Solve the equations.

**11.** (6pts)  $27^{2x+1} = 3^{1-4x}$

**12.** (8pts)  $6^{x-3} = 10^{2x+1}$

**13.** (12pts) Census data has the population of the state of Missouri as 5,989,000 in 2010 and 6,155,000 in 2020. Assume that it has grown according to the formula  $P(t) = P_0e^{kt}$ .

a) Find  $k$  and write the function that describes the population at time  $t$  years since 2010. Graph it on paper.

b) Find the predicted population in the year 2032.

**Bonus** (10pts) Among the four “and” statements below, two are false regardless of which  $a, b, c, d$  are chosen, and two can be true (with the right choice of  $a, b, c, d$ ). Which ones are which and why? (*Hint: use rules for working with logarithms. Note you are not asked to find  $a, b, c, d$ , so don't try. If you can find a reason why the two equations in a statement do not agree with each other, then you have found a false statement.*)

$$\log_a 3 = 0.251 \text{ and } \log_a 9 = 0.502$$

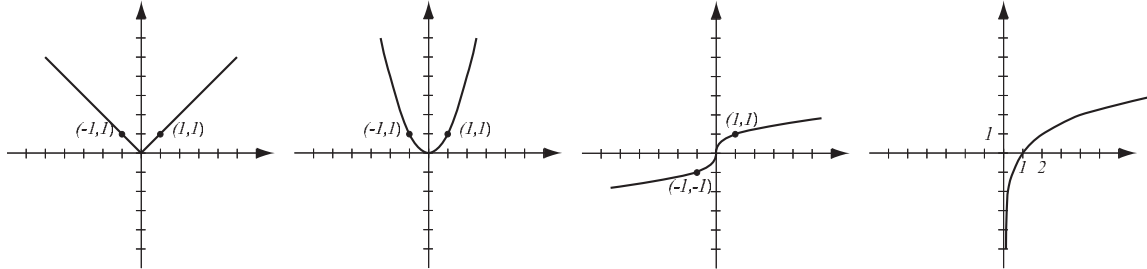
$$\log_b 4 = 1.731 \text{ and } \log_b 16 = 3.03$$

$$\log_c 5 = 1.325 \text{ and } \log_c(5c) = 2.021$$

$$\log_d 7 = 2.513 \text{ and } \log_d(7d) = 3.513$$

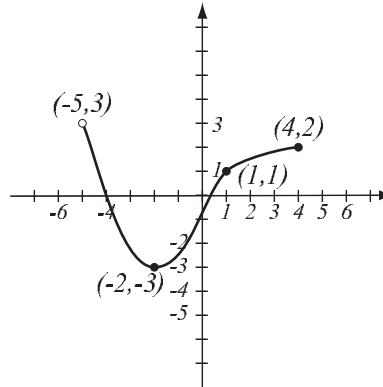


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- Find:  $f(4) =$        $f(-5) =$
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = 1$ ?



3. (5pts) Find the equation of the line that passes through point  $(4, -2)$  and is perpendicular to the line  $y = \frac{2}{3}x - 1$ .

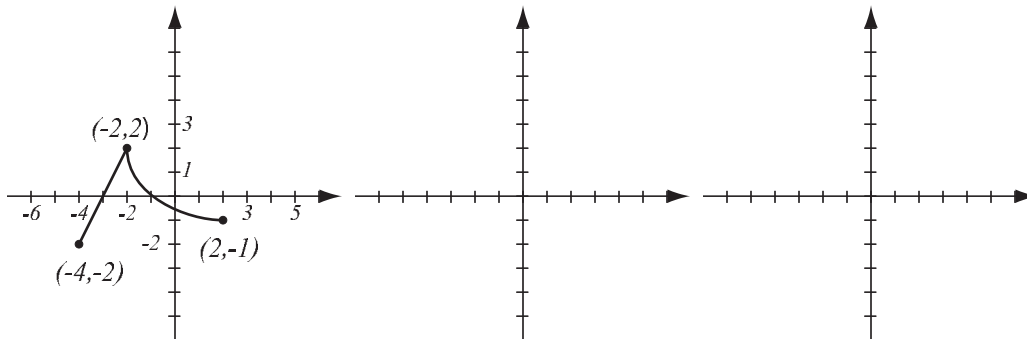
4. (7pts) Draw the triangle with vertices  $A = (-5, 0)$ ,  $B = (5, 0)$  and  $C = (-3, 4)$ . Use either the Pythagorean theorem and lengths of sides or use slopes to determine if this is a right triangle.

5. (3pts) Find the domain of the function  $f(x) = \frac{5}{x^2 - 5}$  and write it in interval notation.

6. (6pts) Solve and write the solution in interval notation.

$$|3x + 1| < 2$$

7. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $2f(x - 2)$  and  $f(-x) - 2$  and label all the relevant points.



8. (6pts) Let  $f(x) = 46e^{x+7}$ . Find the formula for  $f^{-1}$ .

**9.** (12pts) The quadratic function  $f(x) = x^2 + 2x - 8$  is given. Do the following without using the calculator.

a) Find the  $x$ - and  $y$ -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

**10.** (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 (81x^2 \sqrt[6]{y}) =$$

**11.** (6pts) Write as a single logarithm. Simplify if possible.

$$4 \ln(u^{-3}v^2) - 3 \ln(u^6v^2) =$$

**12.** (6pts) Let  $f(x) = x^2 + 3x + 4$ ,  $g(x) = \sqrt{x - 2}$ . Find the following (simplify where possible):

$$(fg)(x) =$$

$$(f \circ g)(x) =$$

**13.** (20pts) The polynomial  $P(x) = x^2(x - 3)(x + 3)$  is given (answer with 6 decimals accuracy).

a) What is the end behavior of the polynomial?

b) Find all the zeros and their multiplicities. Find the  $y$ -intercept.

c) Determine algebraically whether the function is odd, even, or neither. (Multiply out the factors if you need to.)

d) Use the graphing calculator along with a) and b) to sketch the graph of  $P$  (yes, on paper!).

e) Verify your conclusion from c) by stating symmetry.

f) Find all the turning points (i.e., local maxima and minima).

**14.** (8pts) Solve the equation.

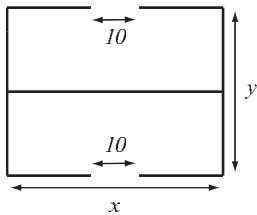
$$x + 9 = 3 - \sqrt{60 + 7x}$$

**15.** (14pts) Kurt drives to Hopkinsville on the highway at 68mph. Due to flooding of the highway, on the way back he has to take a slower route where he averages 54mph. This route is 11 miles longer and takes him 20 minutes more to drive.

- a) How long did Kurt drive to Hopkinsville?
- b) How long was the slower route?

**16.** (14pts) Jeffrey is designing a combo gas station / fast food restaurant building with 10-foot wide entrances on either side. His budget allows for 280 feet of total wall length. Jeffrey's goal is to maximize the total area of the building.

- a) Express the total area of the building as a function of the length of one of the sides. What is the domain of this function?
- b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible total area, and what is the biggest possible total area?



17. (12pts) Census data has the population of the state of Missouri as 5,595,000 in 2000 and 6,155,000 in 2020. Assume that it has grown according to the formula  $P(t) = P_0e^{kt}$ .

a) Find  $k$  and write the function that describes the population at time  $t$  years since 2010. Graph it on paper.

b) Find the predicted population in the year 2027.

**Bonus** (10pts) The graph below was obtained by transformations of a graph of a standard function. Identify the standard function and the transformations and use them to write the formula for the function in the picture.

